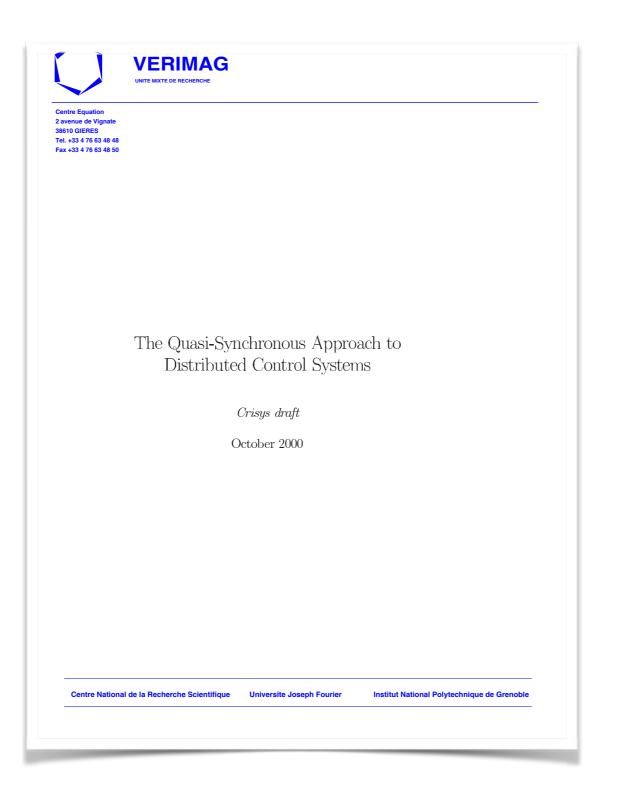
Soundness of the **Quasi-Synchronous Abstraction**

Guillaume Baudart

Timothy Bourke Marc Pouzet

École normale supérieure, INRIA Paris



Chapter 2

The Architectural Evolution

Aircraft control systems illustrate this evolution which can also be found in many other fields of industrial control

2.1 Analog/Digital Communication

Starting from networks of analog boards, progressively some boards were replaced by discrete digital boards, and then by computers. Communication between the digital parts and the parts which remained analog was mainly based on periodic sampling (analog to digital conversion) and holding (digital to analog conversion), sampling periods being adapted to the frequency properties of the signals that traveled through the network. This allowed several technologies to smoothly cooperate. Figure 2.1 illustrates this evolution.

2.2 Serial Links

This technique was suitable up to the time when two connected analog boards were replaced by digital ones. Then these two also had to communicate and serial port communication appeared as the simplest way of replacing analog to digital and digital to analog communication as both can be seen as latches or memories. Figure 2.2 shows a typical situation borrowed from an automatic subway application. Each computer monitors a rail track section and runs a periodic program. Computers are linked together by serial lines

5

8

Crisys Esprit Project

2.4 Supervision

In most cases, this architecture is being added a supervisor, for monitoring purposes. The communication between the supervisor and field computers is however very different from the communication between field computers. It is an event-based communication which is assumed not to be time nor safety critical and which takes place either on special time slots of the field bus, or on a dedicated communication medium. The important fact, here, is that it should not perturbate neither the periodic behavior of field computers, nor their communication.

2.5 Provision against Byzantine Problems

In these very critical systems, Byzantine faults cannot be neglected and this is why some architectural precautions have to be taken in order to alleviate their consequences. For instance, these busses provide some protection against Byzantine problems [22], in the sense that they are based on broadcast: communication with several partners only involve one emission. Thus a failed unit cannot diversely lie to its partners. Then messages are protected by either error correcting and/or detecting codes which can be assumed to be powerful enough so that their failing be negligible with respect to the probabilistic fault tolerance requirements of the system under consideration.

2.6 Communication Abstraction

According to what precedes, we can quite precisely state an abstract property of this kind of communication medium, which is a bounded delay communication property:

Property 1. First, we assume that every process P is periodic with a period varying between small margins:

 $T_{Pm} \leq T_P \leq T_{PM}$

Then,

Property 2. Let T_{sM} and T_{rM} be the respective maximal periods of the sender and of the receiver, and n the maximum number of non negligible consecutive failed receives (in the case of error correction, n = 1).

	8 Crisys Esprit Project
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Chapter 3

A Synchronous Tool set for Quasi-Synchronous Systems

In this chapter we show how synchronous design tools allow a global system description, simulation and validation. We first describe our notation. Then we show how to describe systems implemented on a quasi-synchronous architecture and how to simulate them.

3.1 Synchronous Data-Flow Notations

In the sequel, algorithms are expressed using a functional notation, that is to say by abstracting over time indices, in order to stay consistent with design tools. Thus, a signal definition:

 $x_1 = x_2$ means $\forall n \in N : x_1(nT) = x_2(nT)$.

3.1.1 Usual Operators

An operation:

 $(x_1 - x_2)(nT)$ means $x_1(nT) - x_2(nT)$

and similarly,

11

Quasi-Synchronous Approach

15

3.2.1 Shared Memory

Given a sequence u written in the shared memory at clock cw and an initial content v, the current content of the memory can be expressed as:

mem(v, cw, u) = v fby (current(v, cw) u)

where the delay accounts for short^2 undetermined transmission delays. Then, the sequence read at clock cr is :

u' = mem(v, cw, u) when cr

3.2.2 Formalizing Periodic Clocks

This could be done in some real-time framework, such as timed automata [2], but, for the sake of simplicity, we prefer here to characterize the fact that two independent clocks have approximately the same period by saying that:

Any of the two clocks cannot take the value "t"more than twice between two successive "t" values of the other one.

This can be formalized by saying that the boolean vector stream composed of the two clocks should never contain the subsequence:

$\left[\begin{array}{c}t\\-\end{array}\right]\cdot\left[\begin{array}{c}f\\f\end{array}\right]^*\cdot\left[\begin{array}{c}t\\f\end{array}\right]\cdot\left[\begin{array}{c}f\\f\end{array}\right]^*\cdot\left[\begin{array}{c}t\\-\end{array}\right]$

nor the one obtained by exchanging coordinates. (Here, - is a wild card representing any of the two values $\{t,f\}.)$

Now, such regular expressions yield finite state recognizability and can be associated a finite-state recognizing dynamic system $Same_Period_2^3$.

Furthermore, replacing in what precedes 2 by n allows defining similar $Same_Period_n$ systems.

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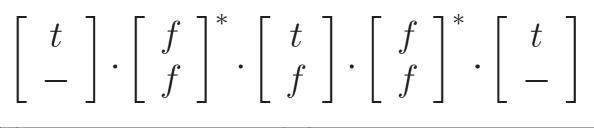
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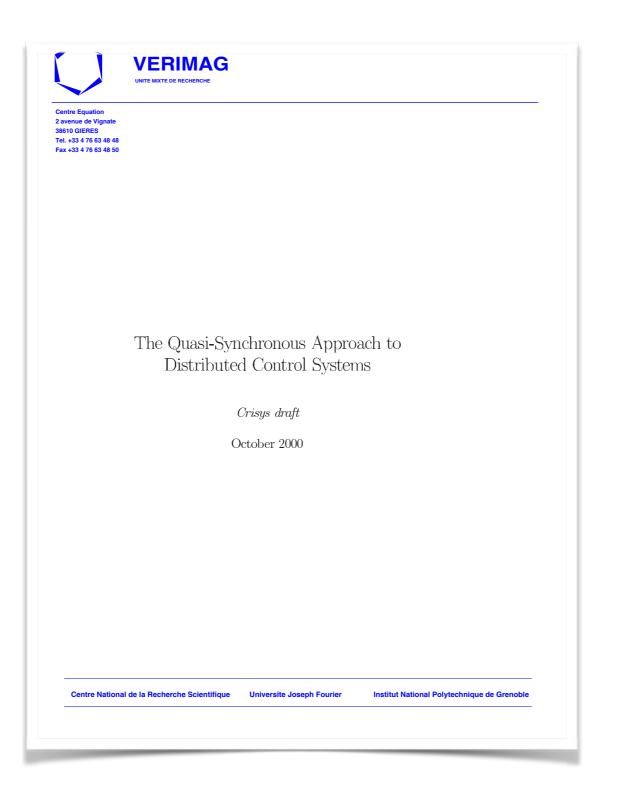
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Quasi-Synchrony

the delay accounts for short² undetermined transmission delays.

²Significantly shorter than the periods of read and write clocks. If longer transmission delays are needed, modeling should be more complex.





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Simulation and Verification of Asynchronous Systems by means of a Synchronous Model*

> Nicolas Halbwachs and Louis Mandel[†] Vérimag[‡], Grenoble – France

Abstract

Synchrony and asynchrony are commonly opposed to each other. Now, in embedded applications, actual solutions are often situated in between, with synchronous processes composed in a partially asynchronous way. Examples of such intermediate solutions are GALS, quasi-synchronous periodic processes, deadline-driven task scheduling...In this paper, we illustrate the use of the synchronous paradigm to model and validate such partially asynchronous applications. We show that, through the use of sporadic activation of processes and simulation of nondeterminism by the way of auxiliary inputs, the synchronous paradigm allows a precise control of asynchrony. The approach is illustrated on a real case study, proposed in the framework of the European Integrated project "Assert".

1 Introduction

It is well admitted, now, that the synchronous paradigm [4, 20] can significantly ease the modeling, programming, and validation of embedded systems and software. The synchronous parallel composition helps in structuring the model, without introducing non-determinism. The determinism of the model is also an invaluable advantage for its validation: tests are reproducible, and modelchecking is not faced with the proliferation of states due to non-deterministic interleaving of processes.

It is also recognized that the synchronous paradigm is not the panacea, since it does not directly apply to intrinsically asynchronous situations, such as distributed systems, or applications mixing long tasks and urgent sporadic requests. This is why numerous works (see, e.g., [7] for a synthesis) are devoted to combining synchrony with asynchrony, or to extending the synchronous model towards less

*This work was partially supported by the European Commission under the Integrated Project Assert, IST 004033 [†]email: {Nicolas.Halbwachs, Louis.Mandel}@imag.fr

*Verimag is a joint laboratory of Université Joseph Fourier, CNRS and INPG associated with IMAG. synchronous applications. For instance, "Communicating reactive processes" [11] or "Multiclock Esterel" [10] are extensions of the synchronous language Esterel [8] to cope with non perfectly synchronous concurrency. On the other hand, the paradigm of "Globally asynchronous, locally synchronous systems" (GALS) has been proposed [16, 1, 9] to describe general asynchronous systems, while keeping as much as possible the advantages of synchronous components. "Tag machines" [6, 5] are an even more general and abstract attempt in the same direction. [17] mixes synchronous (Signal) and asynchronous (Promela) models for verifying GALS.

Another track of research addresses the compilation of synchronous programs towards distributed or non strictly synchronous code. While some distribution methods aim at strictly preserving the synchronous semantics [13, 12], other proposals only preserve the functional semantics [14, 15, 27, 26].

Finally, other works concern the modeling of asynchronous systems within the synchronous paradigm. It is well-known since [24, 25] that a synchronous formalism can be used to express asynchrony. The only need is to express sporadic activation (or stuttering) of processes — which is allowed in all existing synchronous languages — and explicit non-determinism. The modeling tool Model-Build [2, 3] — developed within the European projects SafeAir and SafeAir2 — and the Polychrony workbench [23, 19, 18] are based on this idea. In this paper, we report on our use of this kind of approach in the framework of the Assert project.

Assert is a European Integrated Project devoted to the design of embedded systems from the system architecture level down to the code, with special emphasis on highlevel modeling, proof-based design, and component reuse. Aerospace industry (avionics, launchers, and satellites) constitutes the main application domain of Assert. In this framework, we propose a methodology based on a highlevel behavioral modeling and verification of an application, using a synchronous formalism. Since the automatic generation of distributed code is not an objective of the project, the automatic code generation is only applied separately to

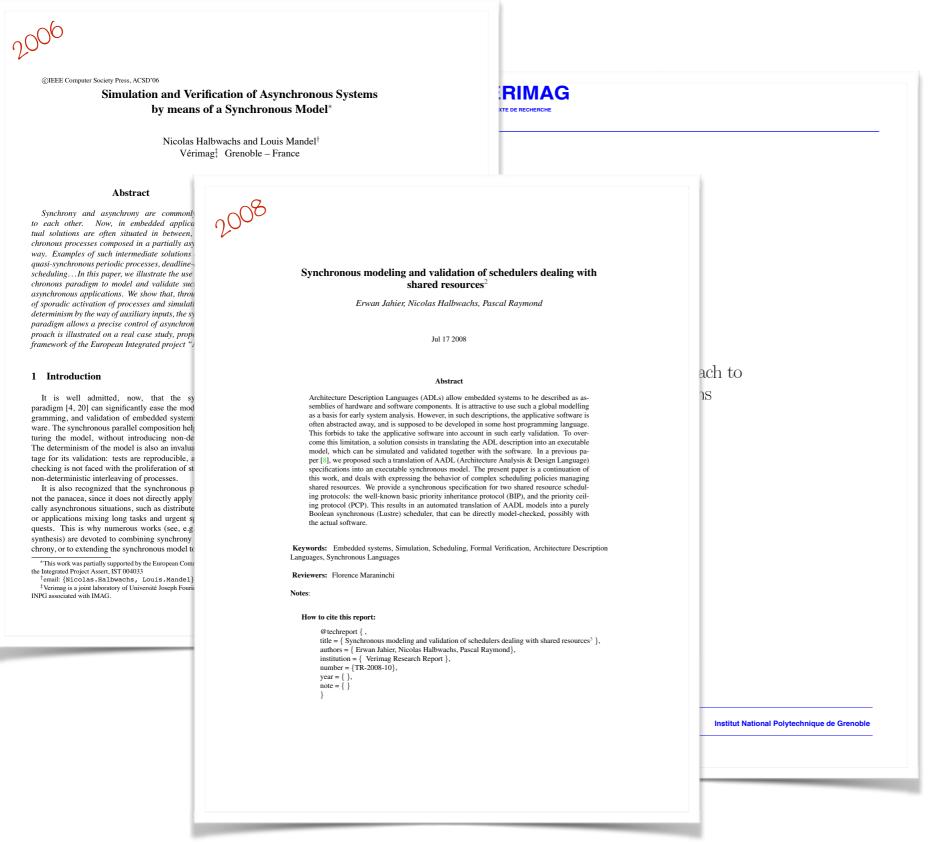
RIMAG

Quasi-Synchronous Approach to Distributed Control Systems

Crisys draft

October 2000

Centre National de la Recherche Scientifique Universite Joseph Fourier Institut National Polytechnique de Gre





per [8], we proposed such a translation of AADL (Architecture Analysis & Design Language) specifications into an executable synchronous model. The present paper is a continuation of this work, and deals with expressing the behavior of complex scheduling policies managing shared resources. We provide a synchronous specification for two shared resource schedul ing protocols: the well-known basic priority inheritance protocol (BIP), and the priority ceiling protocol (PCP). This results in an automated translation of AADL models into a purely Boolean synchronous (Lustre) scheduler, that can be directly model-checked, possibly with the actual software.

Keywords: Embedded systems, Simulation, Scheduling, Formal Verification, Architecture Descrip Languages, Synchronous Languages

Reviewers: Florence Maraninchi

Notes

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VERIFICATION OF OUASI-SYNCHRONOUS SYSTEMS WITH UPPAAL

S. Bhattacharyya⁺, S. Miller⁺, J. Yang⁺⁺, S. Smolka⁺⁺, B. Meng⁺⁺⁺, C. Sticksel⁺⁺⁺, C. Tinelli⁺⁺⁺ ⁺Rockwell Collins, Advanced Technology Center, Cedar Rapids, IA ++ SUNY, Stony Brook, NY +++ University of Iowa, Iowa City, IA

Modern defense systems are complex distributed software systems implemented over heterogeneous and constantly evolving hardware and software platforms. Distributed agreement protocols are often developed exploiting the fact that their systems are quasi-synchronous, where even though the clocks of the different nodes are not synchronized, they all run at the same rate, or multiples of the same rate,

This paper describes an effort to provide systems designers and engineers with an intuitive modeling environment that allows them to specify the highlevel architecture and synchronization logic of quasisynchronous systems using widely available systemsengineering notations and tools. To this end, a translator was developed that translates system architectural models specified in a subset of SysML into the Architectural Analysis and Description Language (AADL). Translators were also developed that translate the AADL models into the input language of the Uppaal and Kind model checkers.

The Uppaal model checker. supports the modeling, verification, and validation of real-time systems modeled as a network of timed automata. This paper focuses on the challenges encountered in translating from AADL to Uppaal, and illustrates the overall approach with a common avionics example: the Pilot Flying System.

Keywords: AADL, quasi-synchronous, model checking, verification, Uppaal, Pilot Flying system.

Introduction

Modern defense systems are complex software systems implemented over heterogeneous and constantly evolving hardware and software platforms. Due to the failure rates of individual hardware components, critical functions must be implemented as redundant, fault-tolerant systems in order to meet

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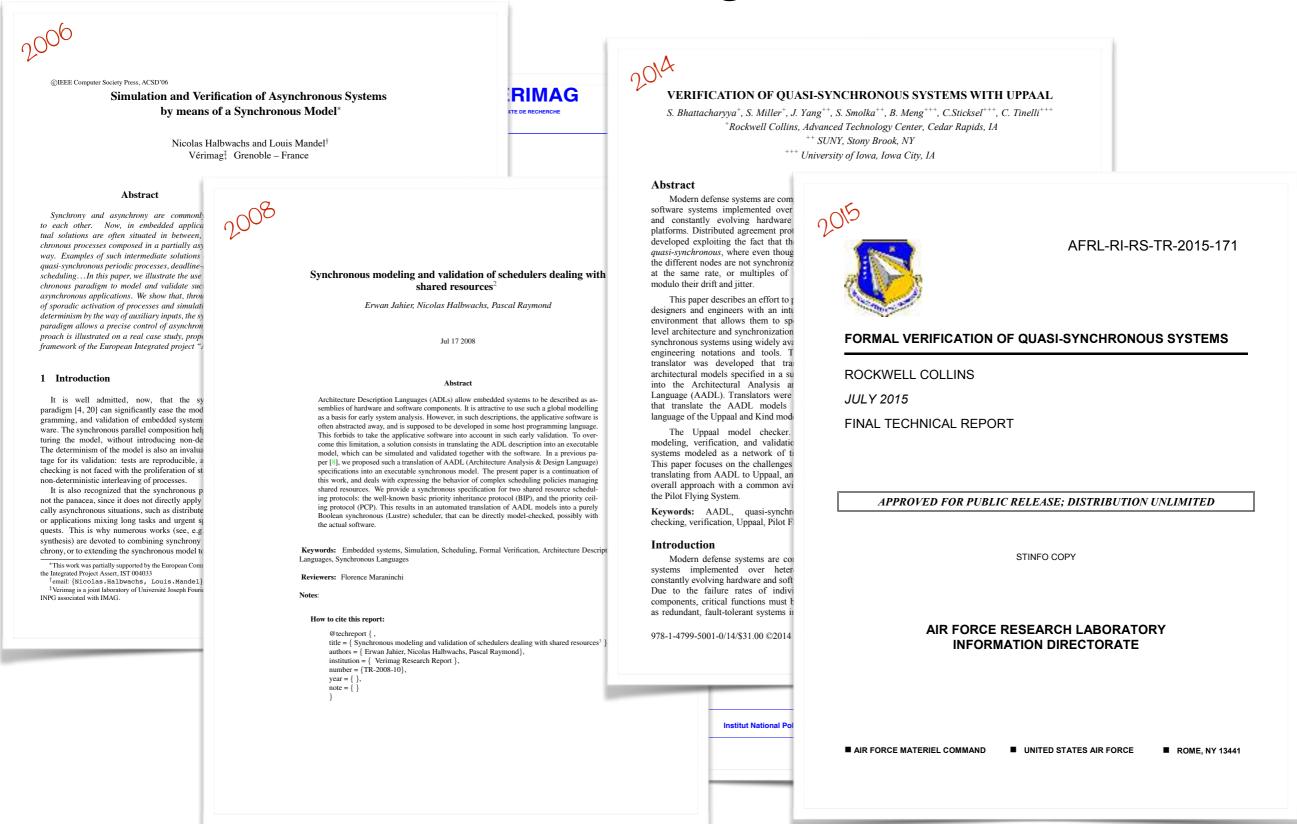
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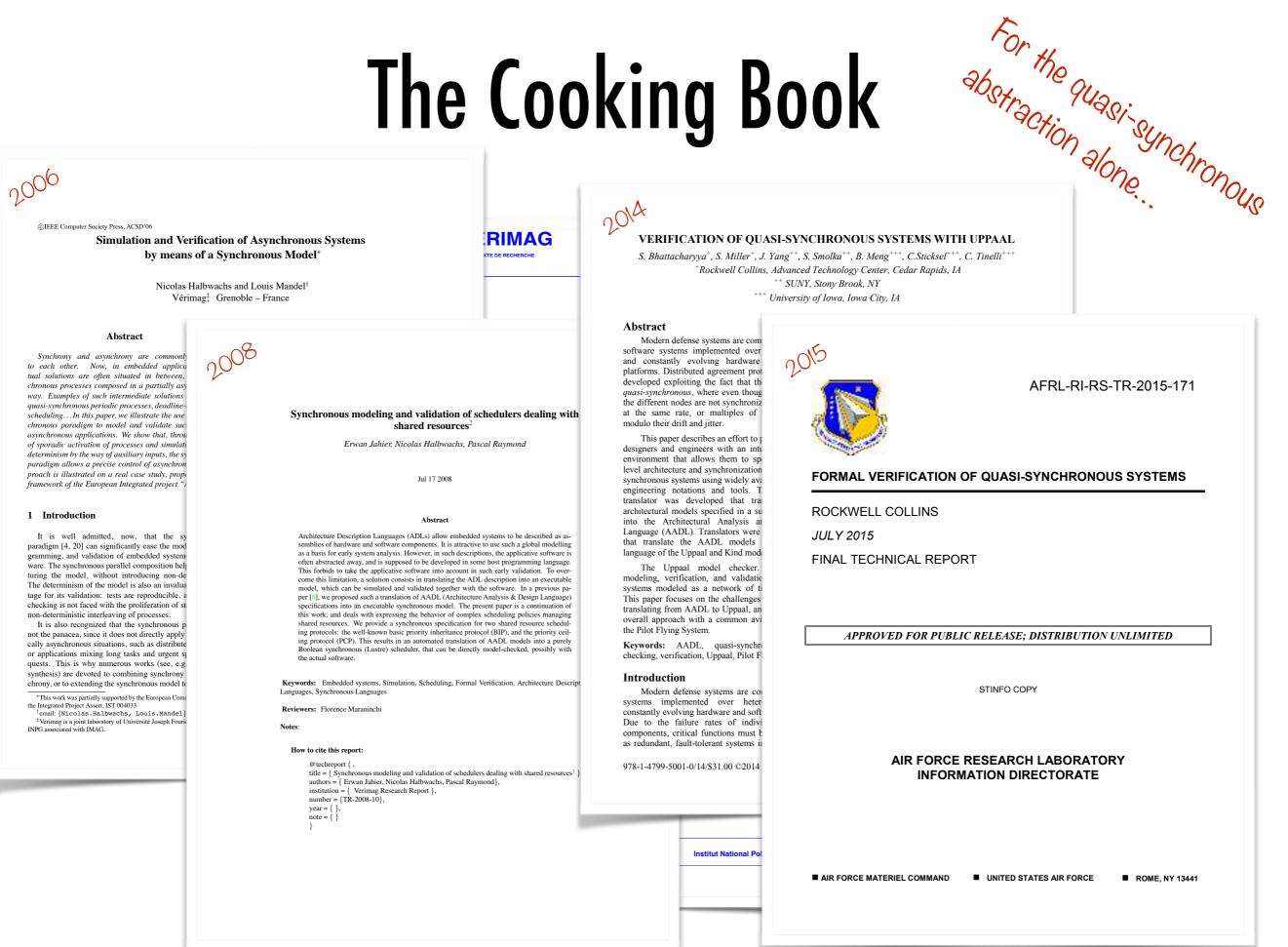
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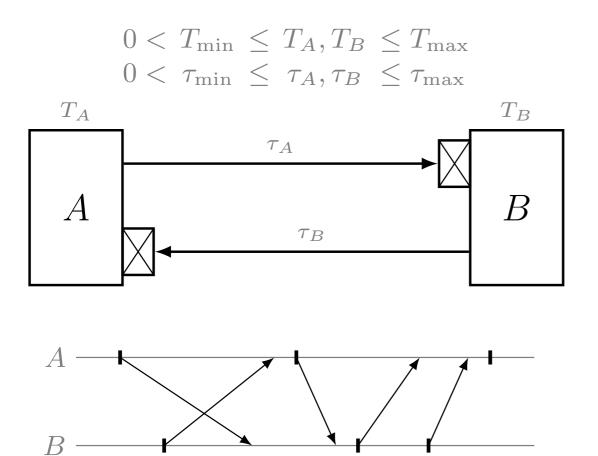
their reliability requirements. This is achieved by distributing these functions over multiple processing components connected by fault-tolerant networks. When a system is replicated to achieve a high level of reliability, the individual components still need to agree on some part of the global system state, such as which node is the current leader. While the amount of state that needs to be consistent is often small, the required consistency is essential for the correct behavior of the system.

In developing distributed agreement protocols, engineers often exploit the fact that their systems are auasi-synchronous, where even though the clocks of the different nodes are not synchronized, they all run at the same rate, or multiples of the same rate, modulo their drift and jitter. While such designs often appear to work correctly at first, their intrinsic asynchrony makes them prone to race and deadlock conditions. These latent design errors often do not appear until late in system integration or even after the system is deployed.

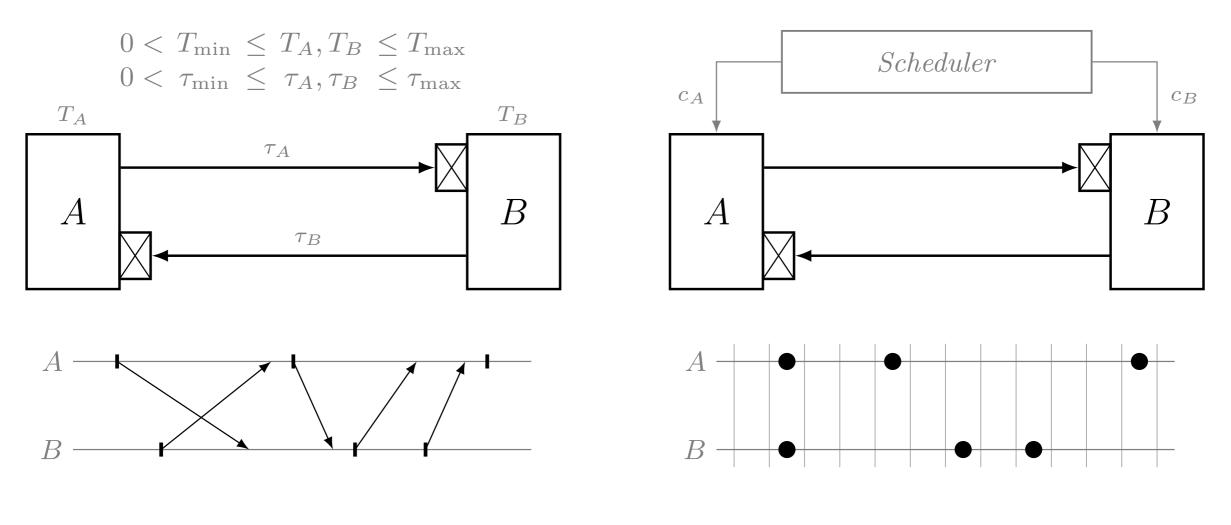
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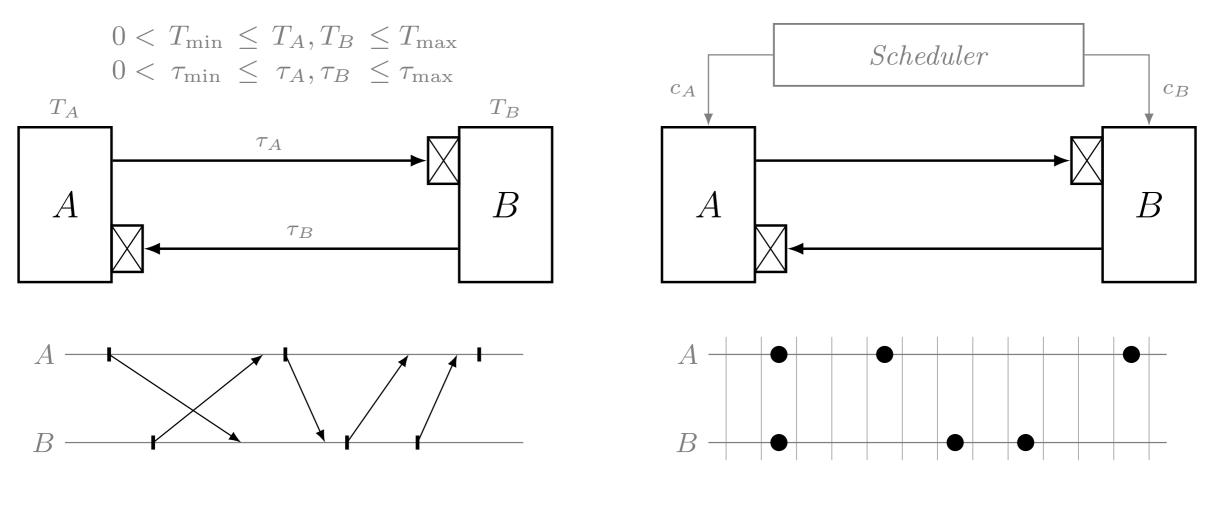


Real-time Model (RT)



Real-time Model (RT)

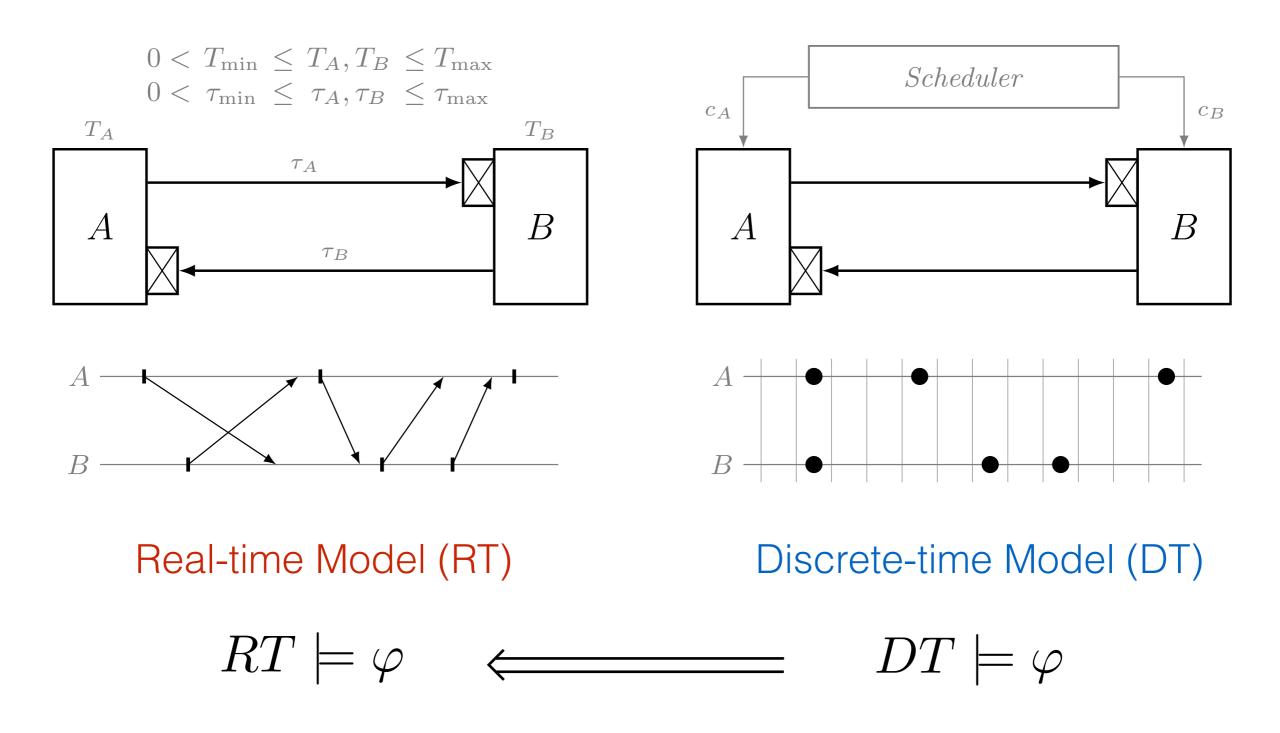
Discrete-time Model (DT)

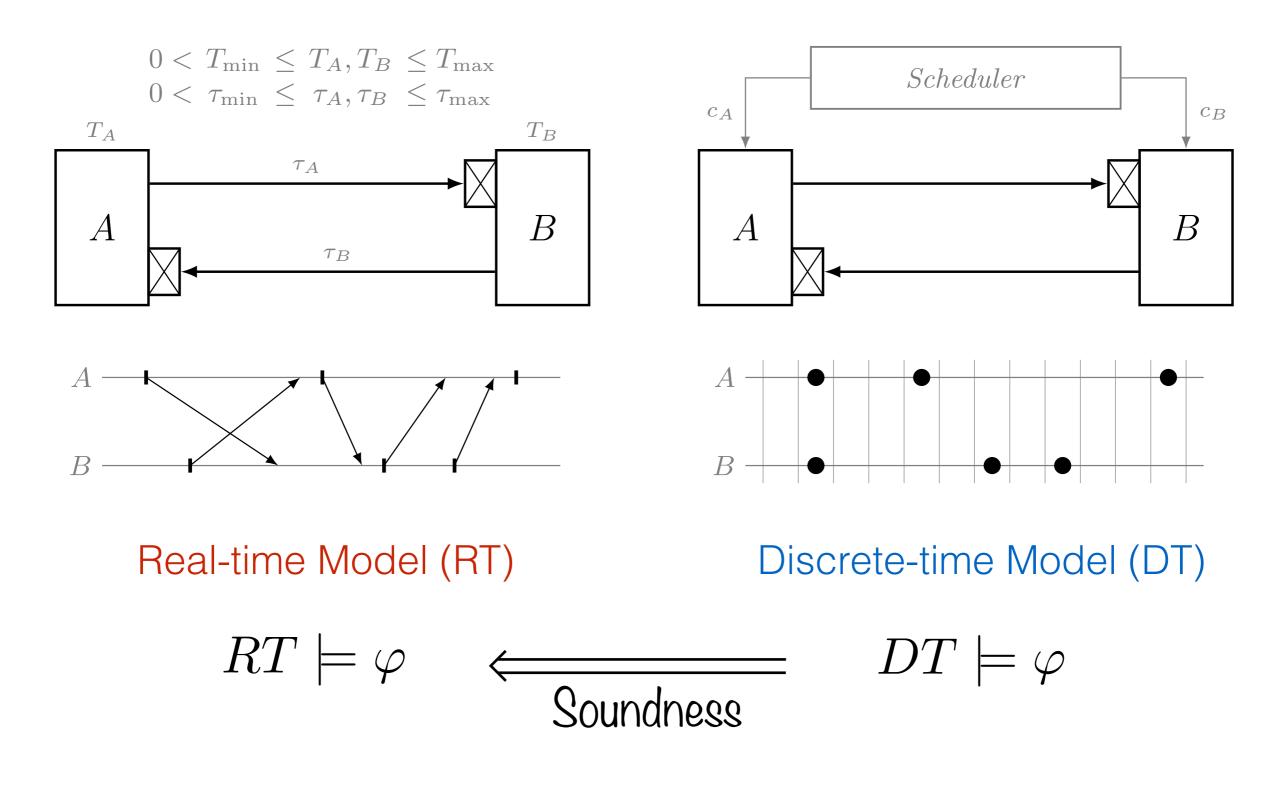


Real-time Model (RT)

Discrete-time Model (DT)

 $DT \models \varphi$





Quasi-Periodic Architectures

Property 1. First, we assume that every process P is periodic with a period varying between small margins:

 $T_{Pm} \le T_P \le T_{PM}$

Definition (Quasi-Periodic Architecture):

• A set of "quasi-periodic" processes with local clocks and nominal period T^n (jitter ε)

 $0 < T_{\min} \le T^n \le T_{\max} \quad \text{or} \\ T^n - \varepsilon \le \kappa_i - \kappa_{i-1} \le T^n + \varepsilon$

$(\kappa_i)_{i\in\mathbb{N}}$ clock activations

- Buffered communication without message inversion or loss
- Bounded communication delay

 $\tau_{\min} \le \tau \le \tau_{\max}$

Quasi-Periodic Architectures

Property 1. First, we assume that every process P is periodic with a period varying between small margins:

 $T_{Pm} \le T_P \le T_{PM}$

Definition (Trace): A (quasi-periodic) trace \mathcal{E} is a set of activation events $\{A_i \mid A \in \mathcal{N} \land i \in \mathbb{N}\}$ and two functions

- $t(A_i)$ the date of event
- $\tau(A_i, B)$ the transmission delay of message sent at A_i to B

For a quasi-periodic trace we have

$$0 < T_{min} \leq t(A_{i+1}) - t(A_i) \leq T_{max}, \\ 0 < \tau_{min} \leq \tau(A_i, B) \leq \tau_{max}.$$

Quasi-Periodic Architectures

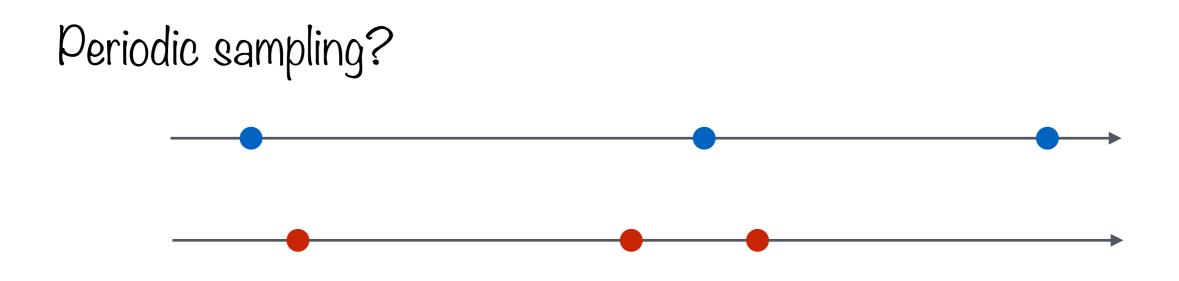
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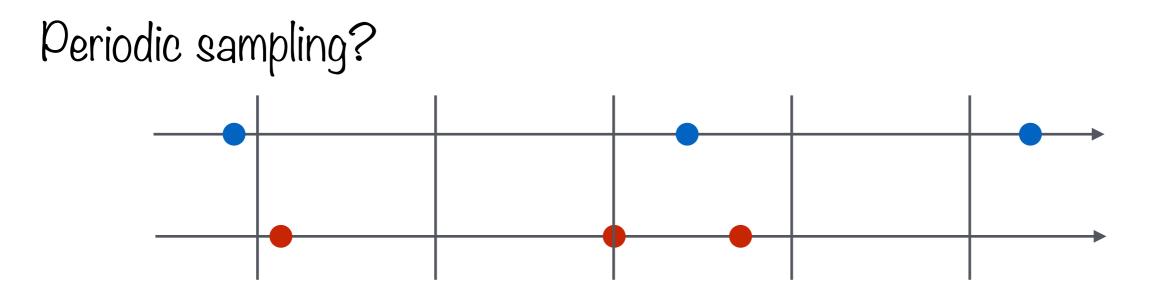
 $T_{Pm} \le T_P \le T_{PM}$

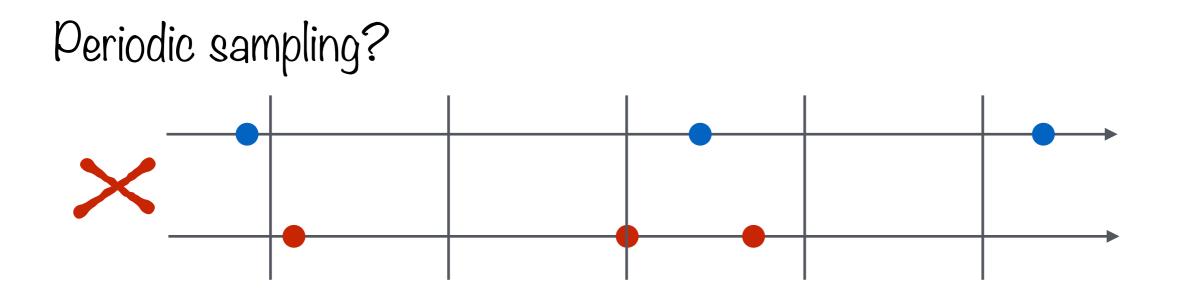
Definition (Happened Before): For a trace \mathcal{E} , let \rightarrow be the smallest relation on activation events that satisfies

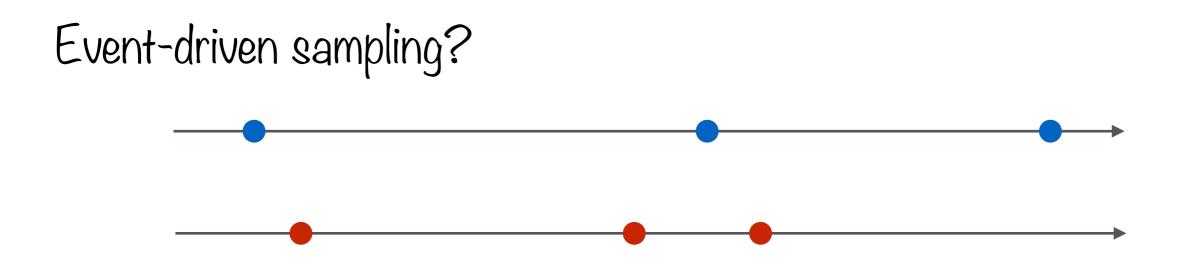
(local) If $i \leq 0, A_i \rightarrow A_{i+1}$ (recv) If $t(A_i) + \tau(A_i, B) \leq t(B_j)$ then $A_i \rightarrow B_j$

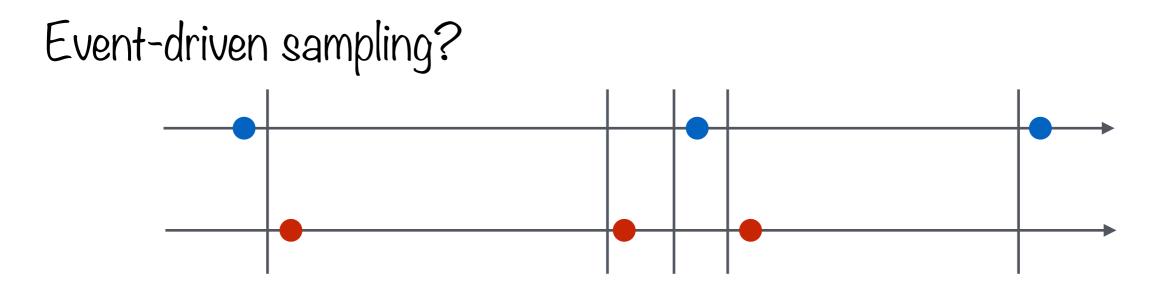
> Node are only triggered by their local clock Message receptions are not explicitly modelled.

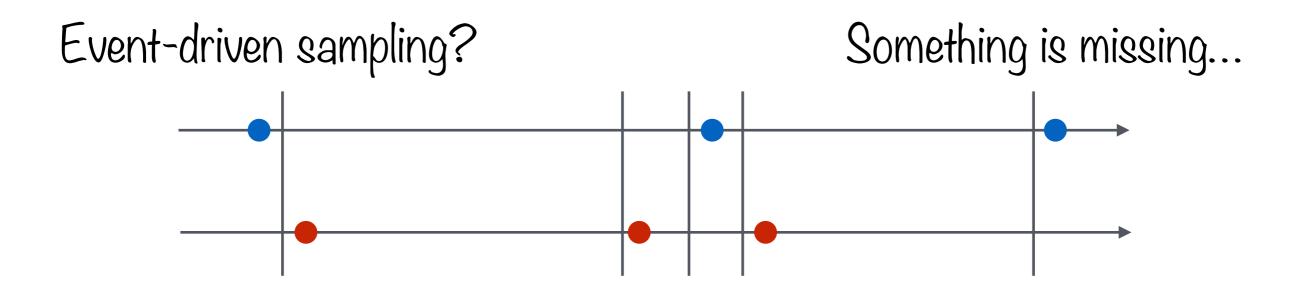




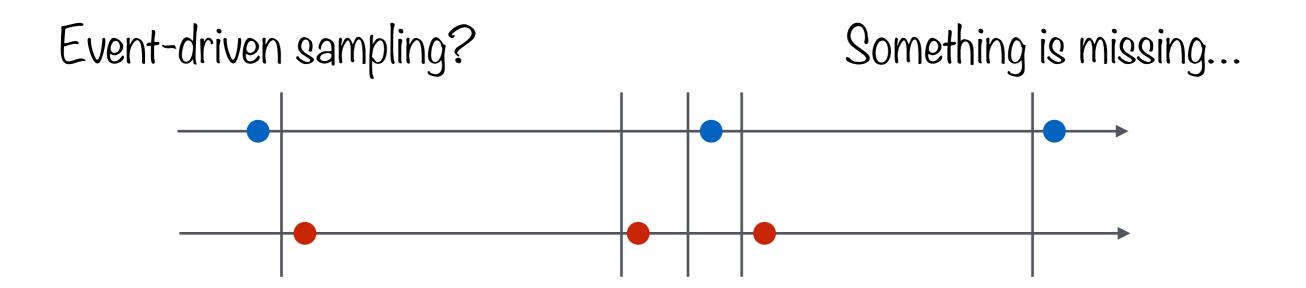




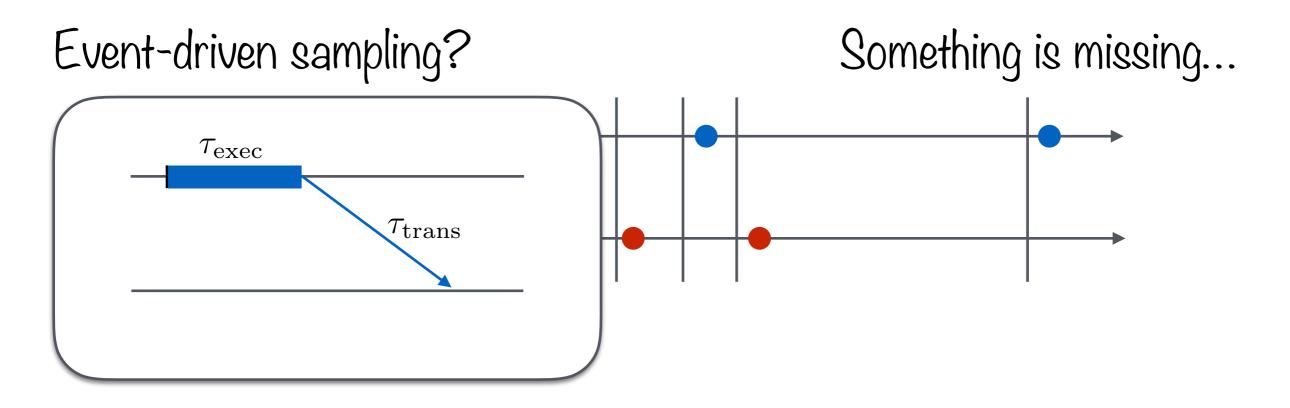




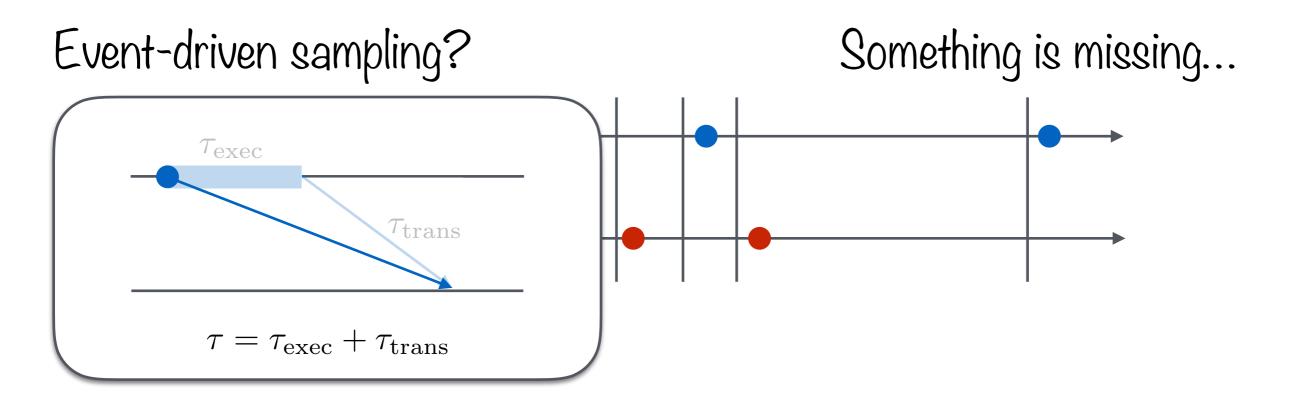
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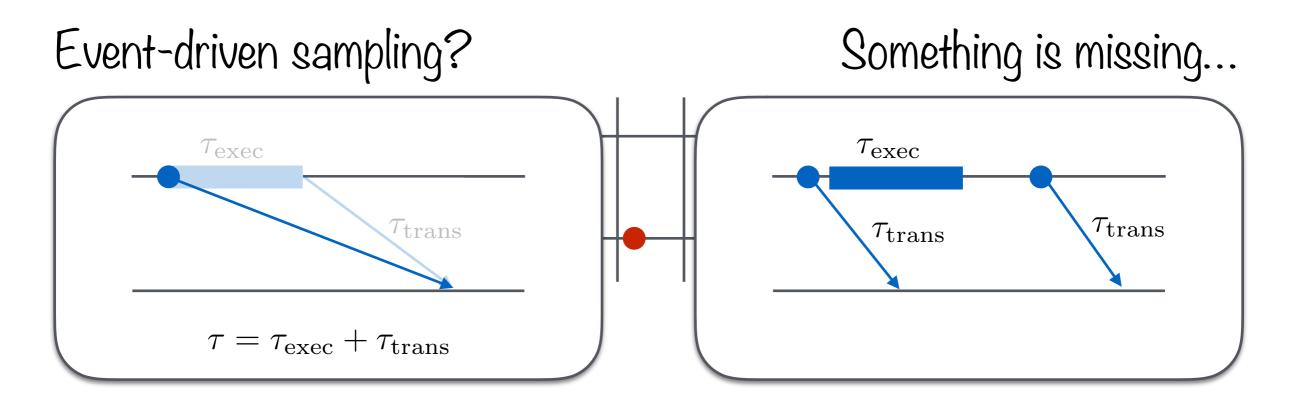
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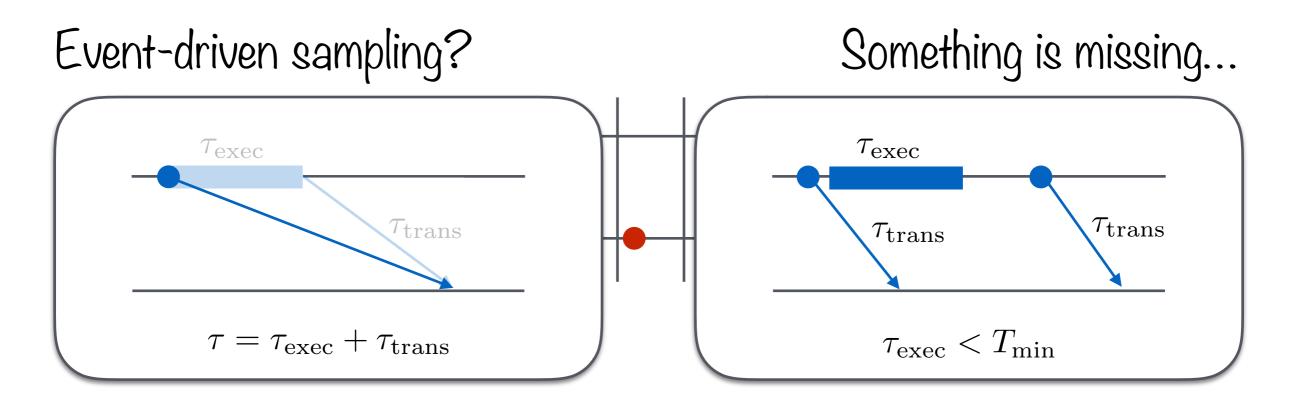
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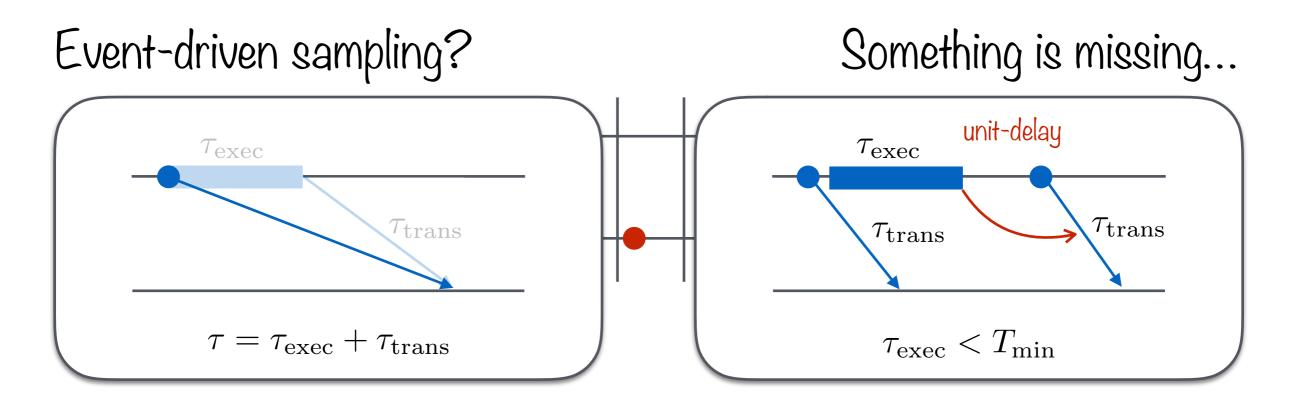
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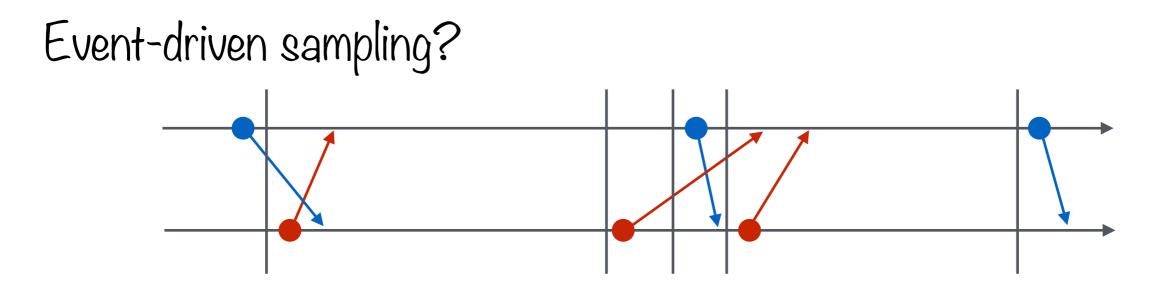
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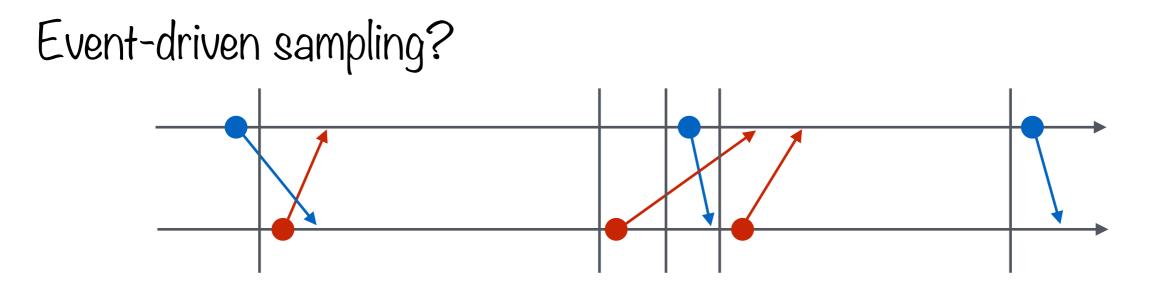
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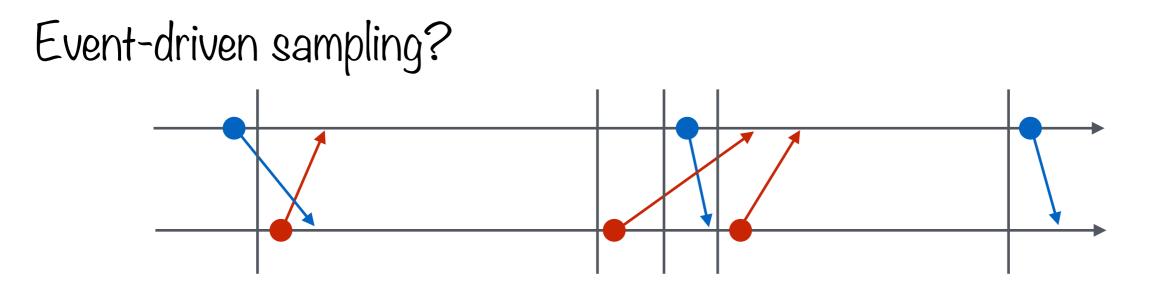


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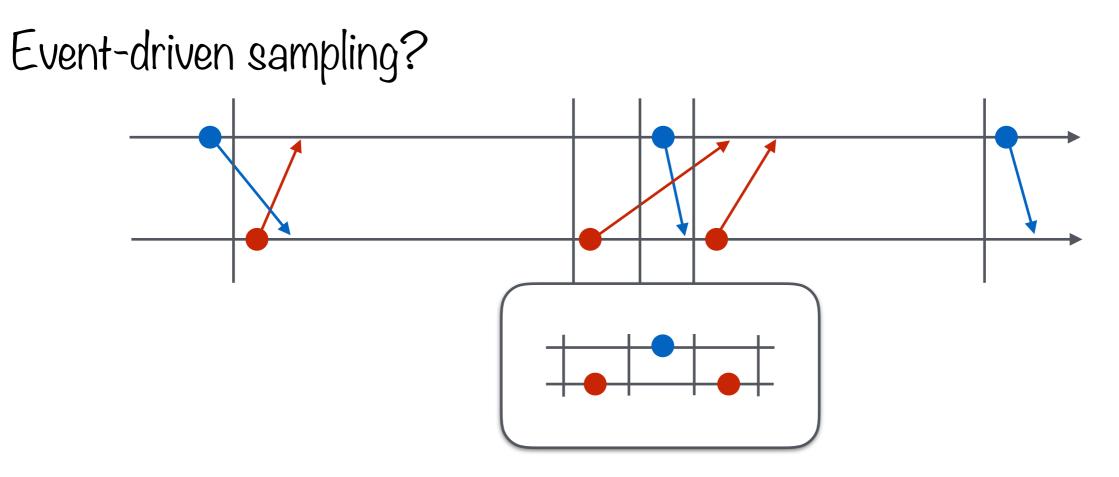
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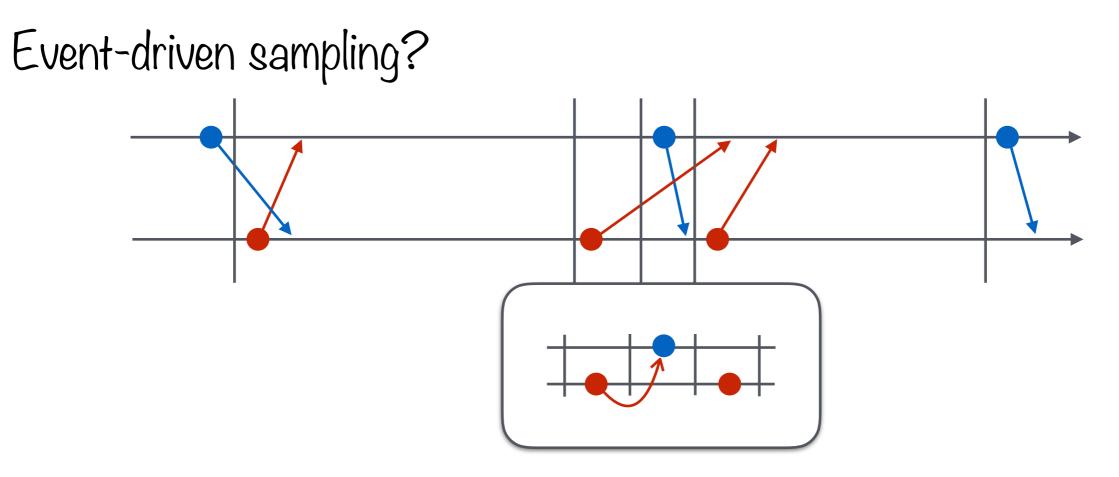
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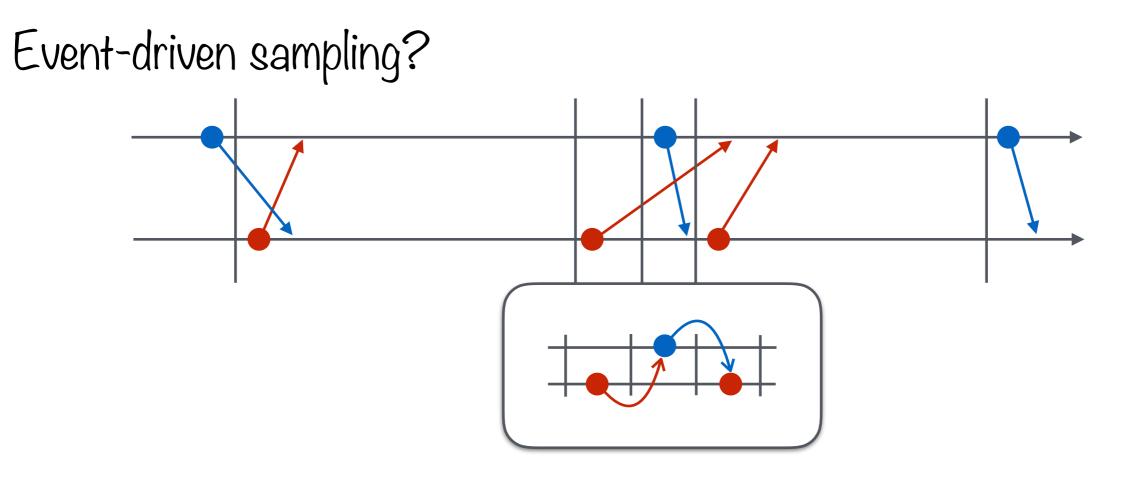
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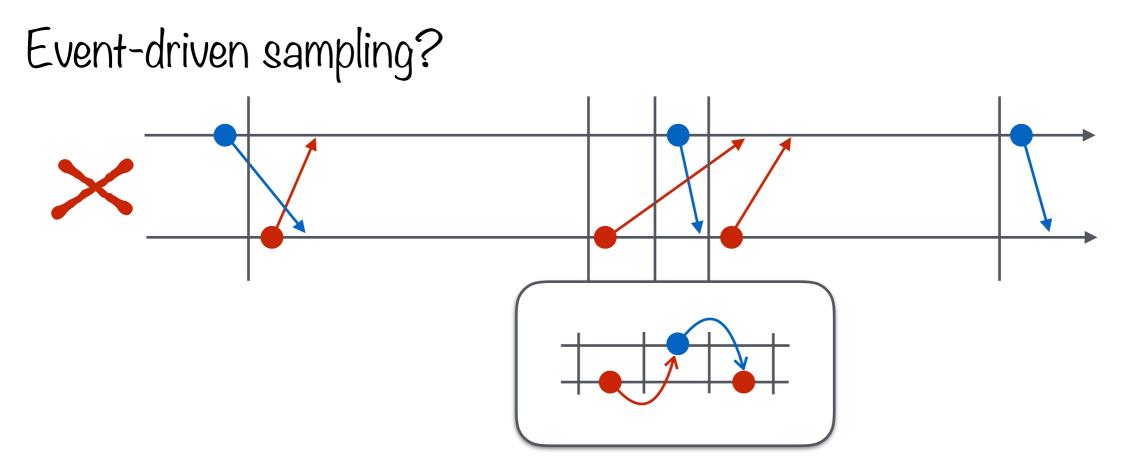
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$$\forall A_i, B_j \in \mathcal{E}, \ A_i \to B_j \iff f(A_i) < f(B_j) \text{ and } A \rightrightarrows B$$

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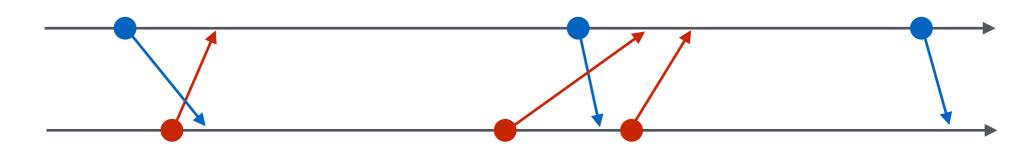
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Problem: Are quasi-periodic architectures unitary discretizable?

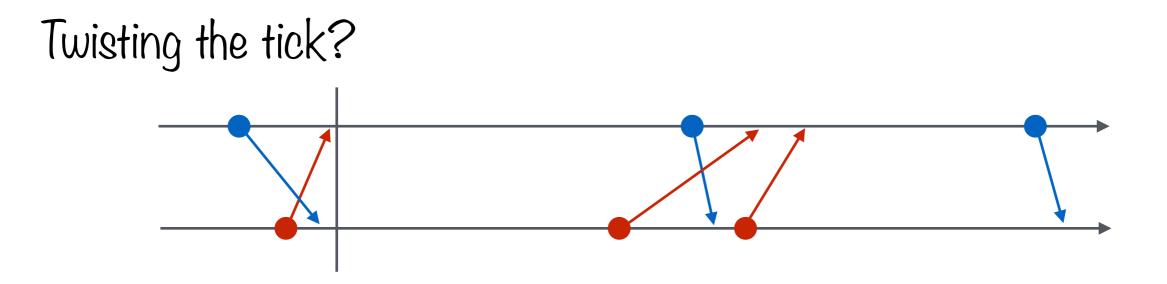
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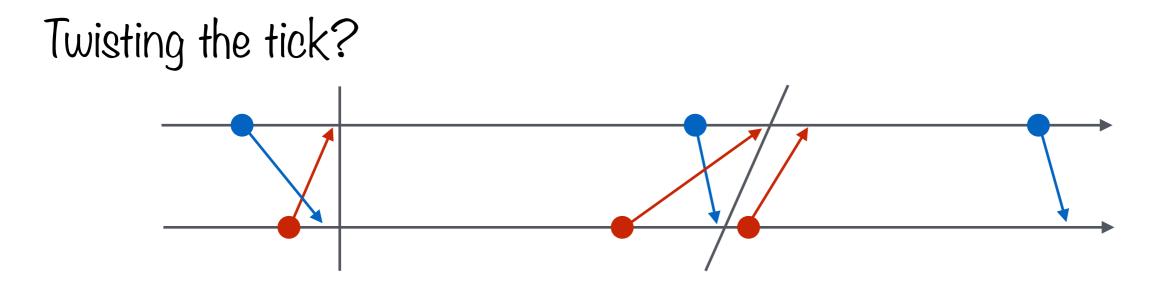
Twisting the tick?



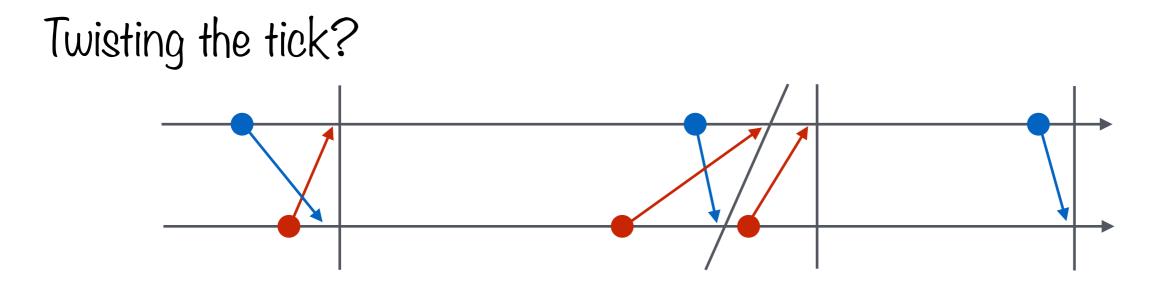
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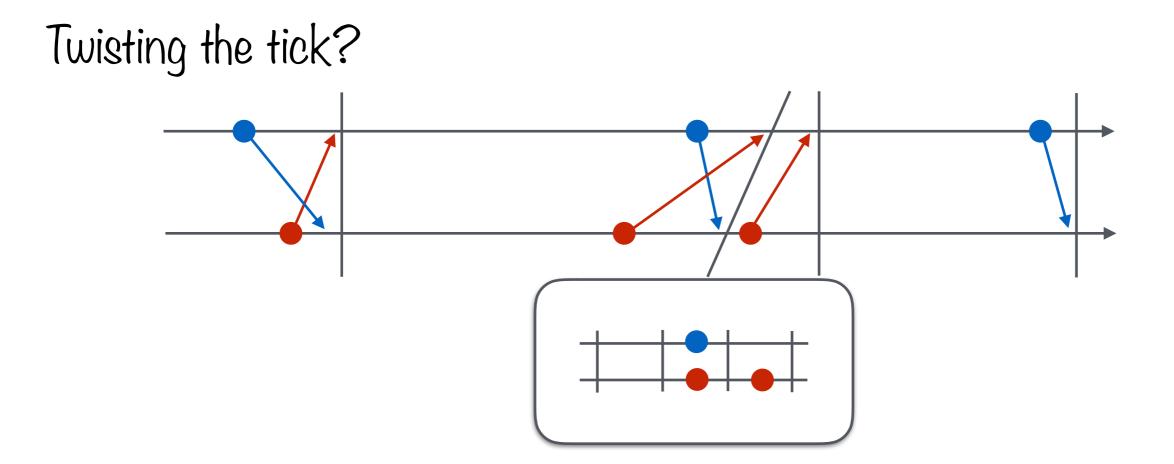
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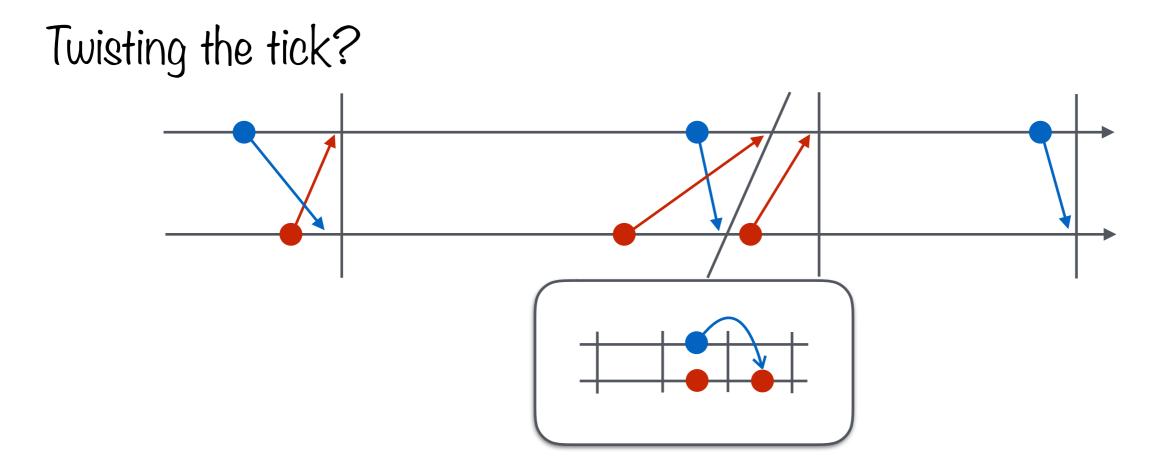
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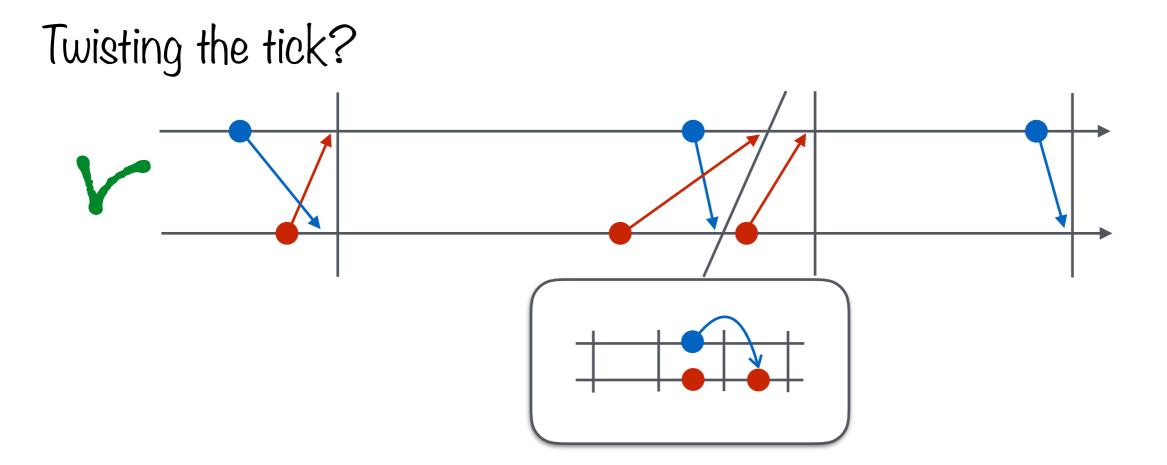
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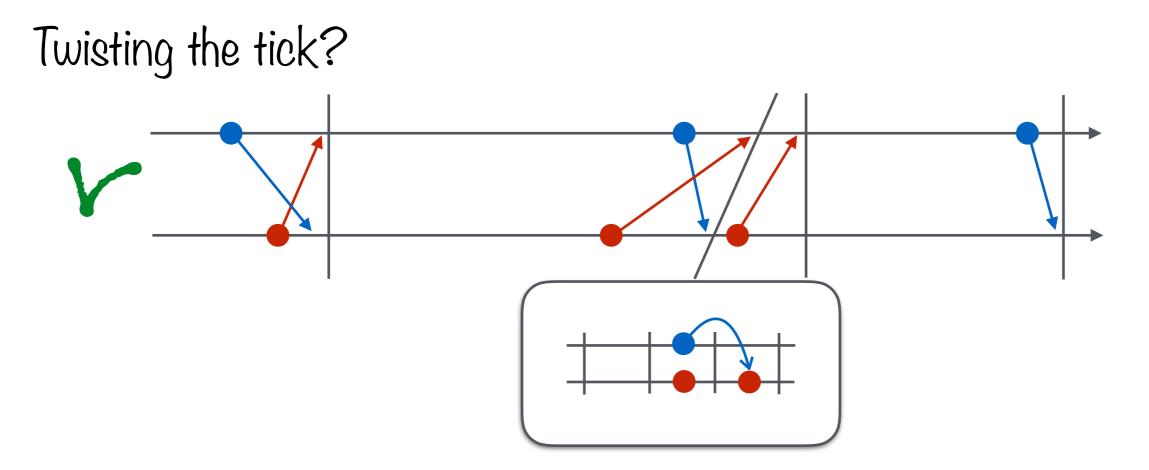


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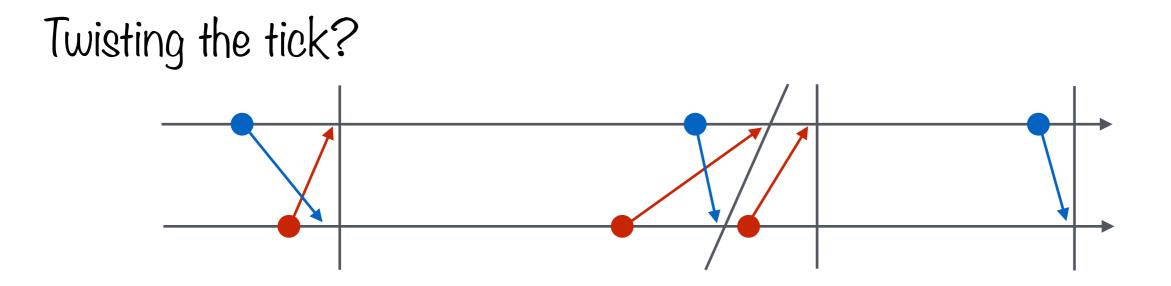
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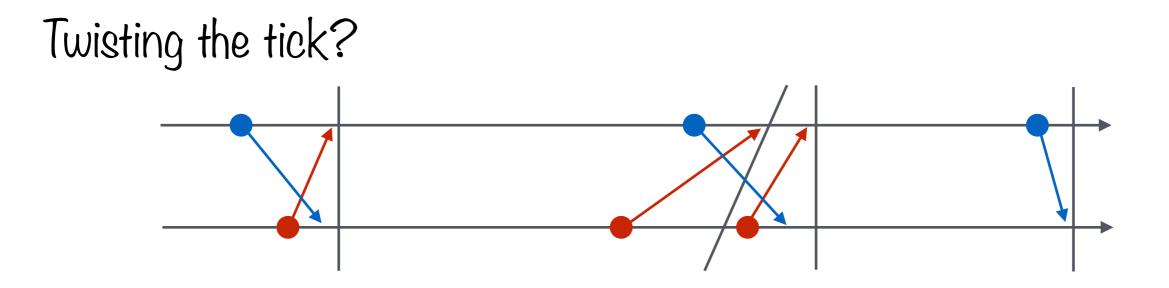


We loose the link between discrete- and real-time

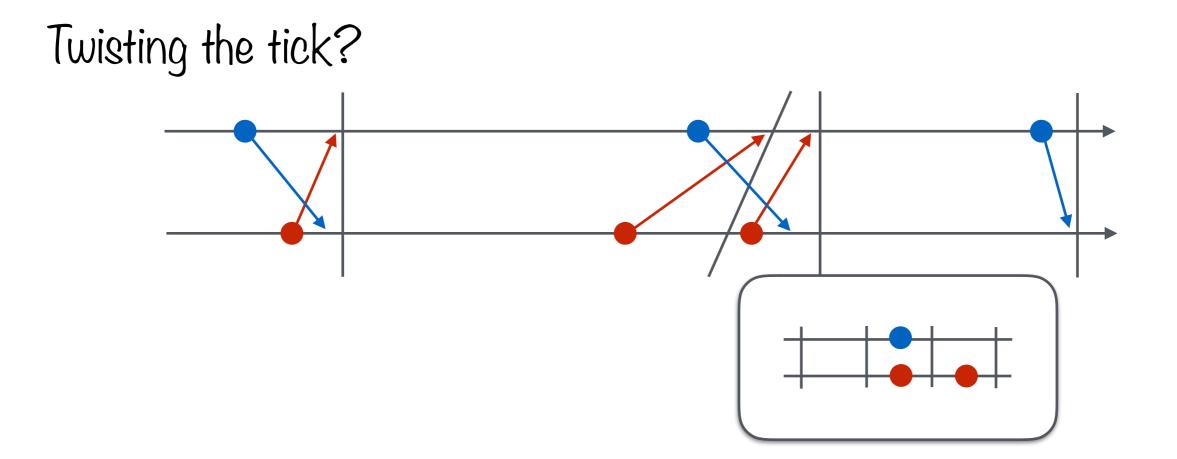
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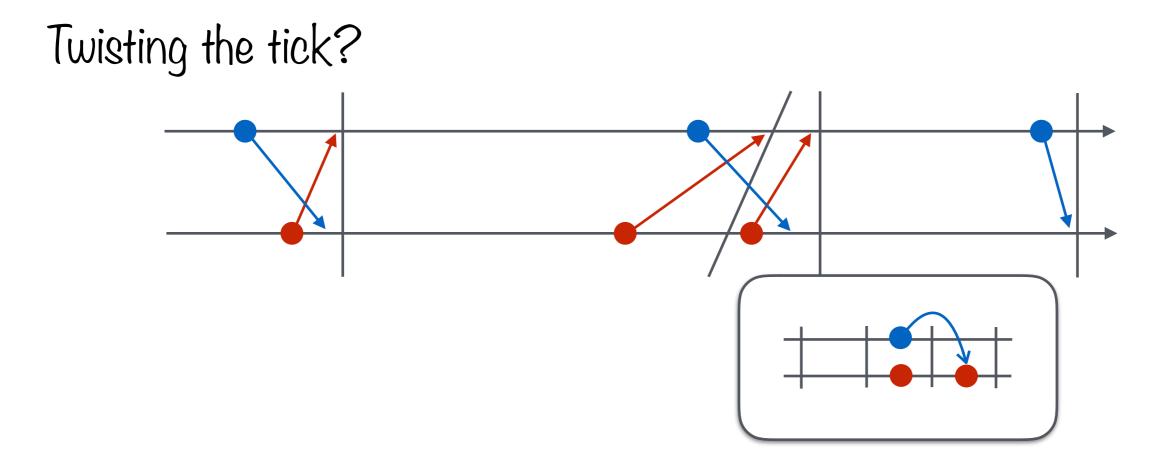
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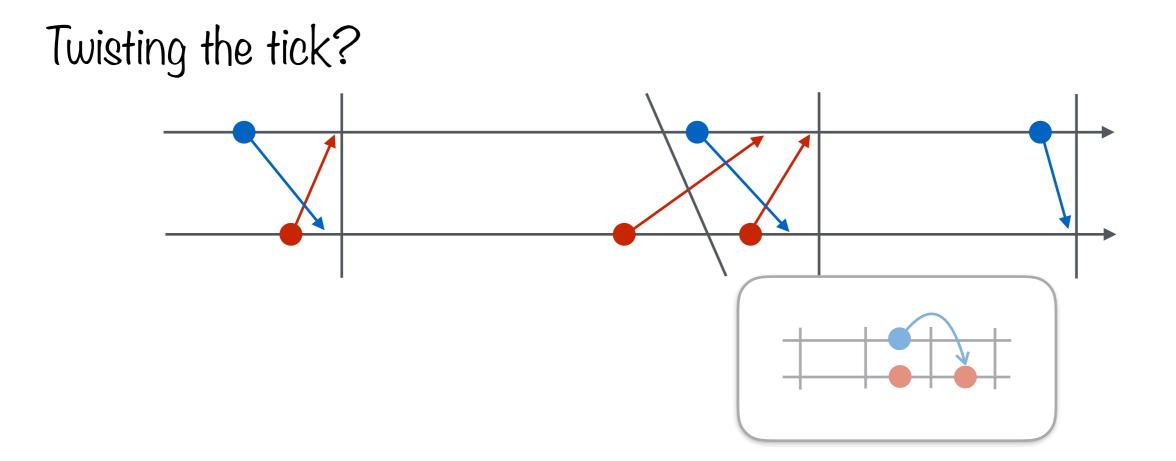
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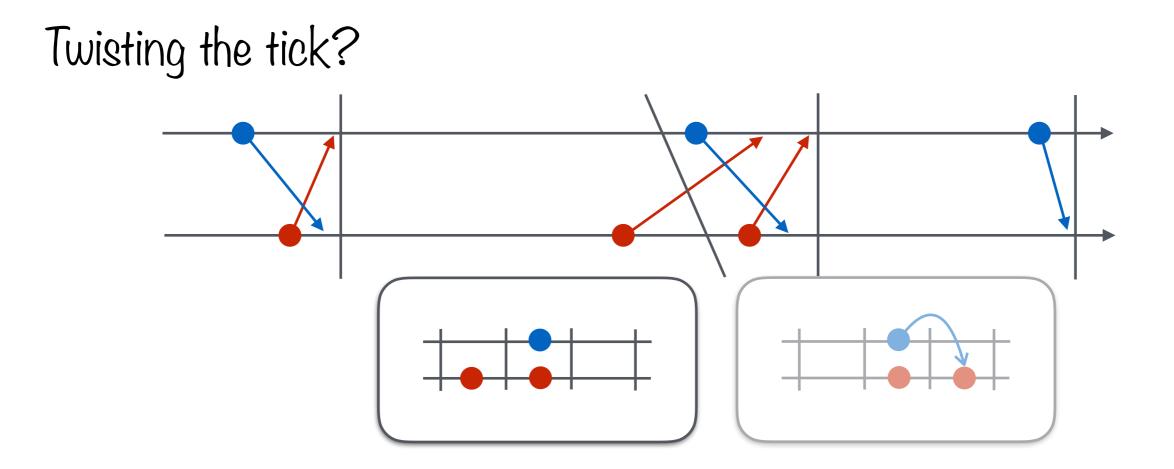
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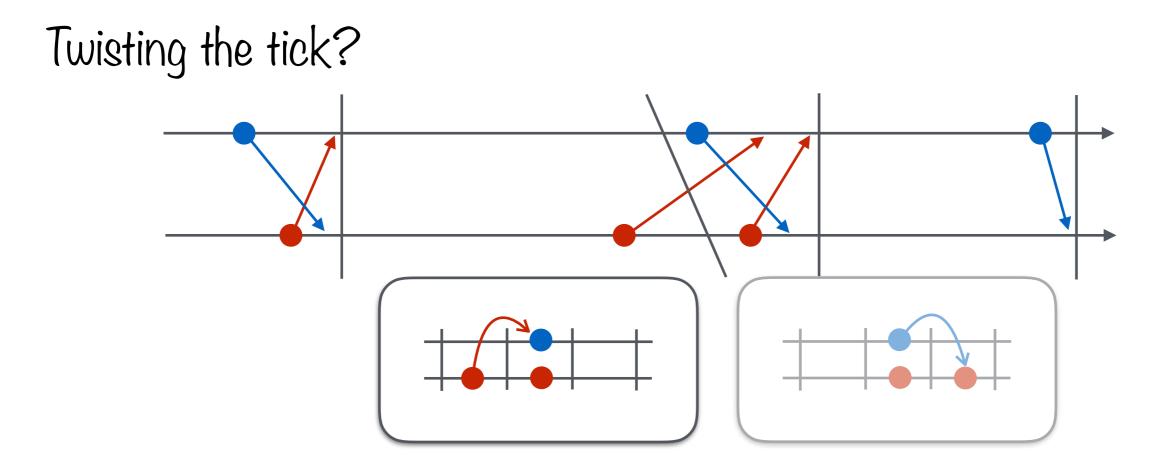
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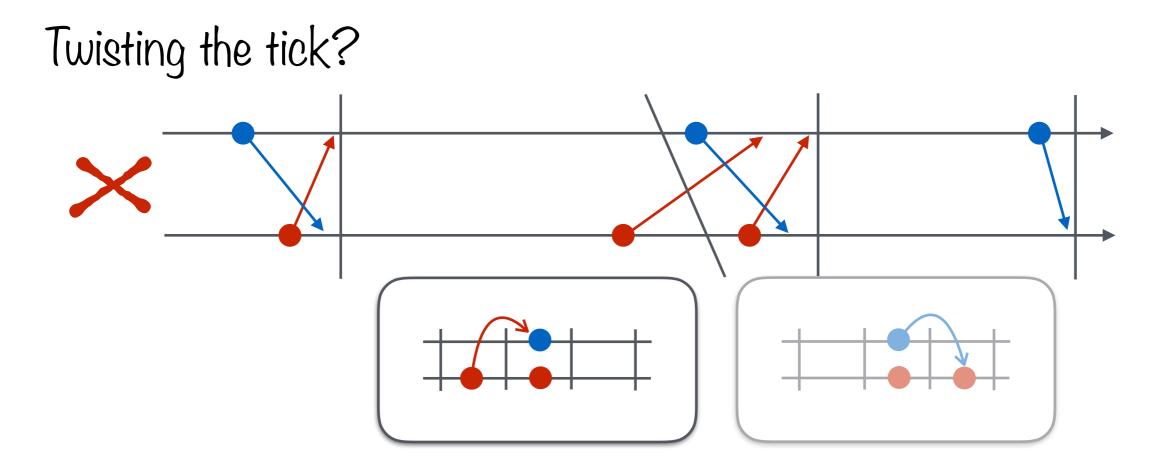
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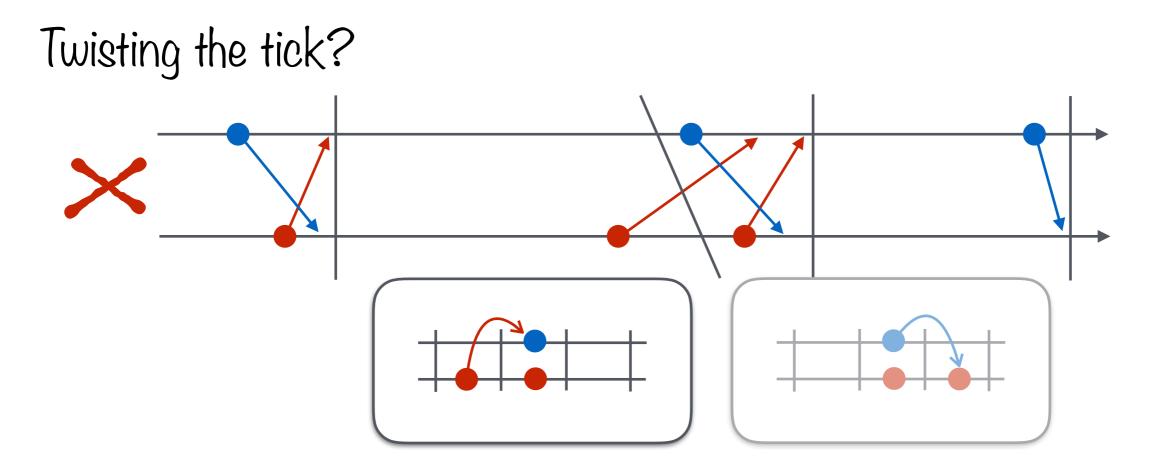


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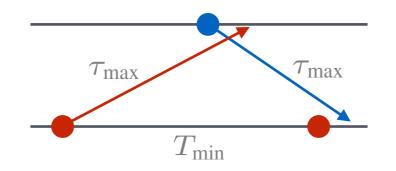
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Some traces cannot be unitary discretized

Theorem (2-nodes systems): A quasi-periodic architectures with two nodes is unitary discretizable if and only if

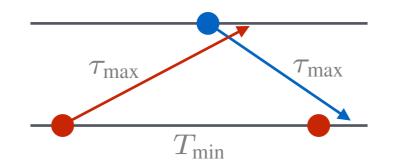
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Worst-case scenario

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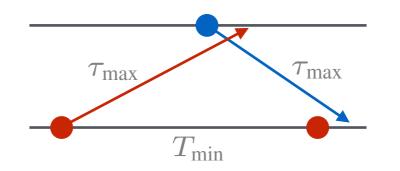


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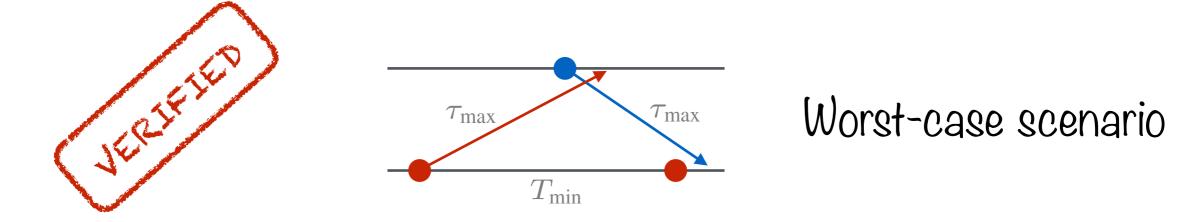


Worst-case scenario

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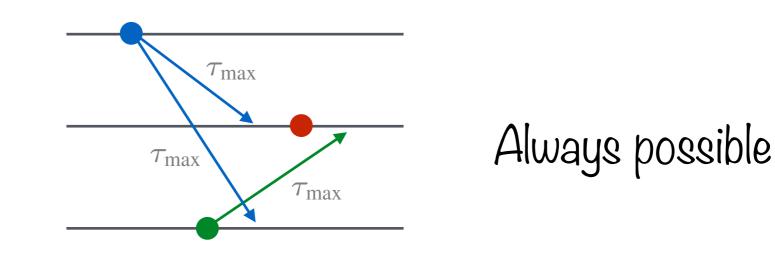
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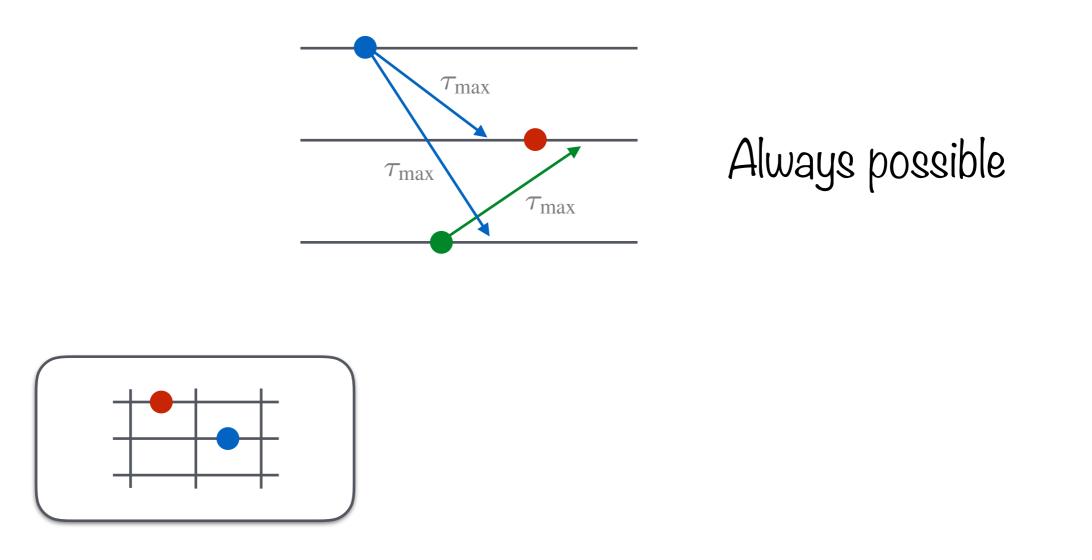


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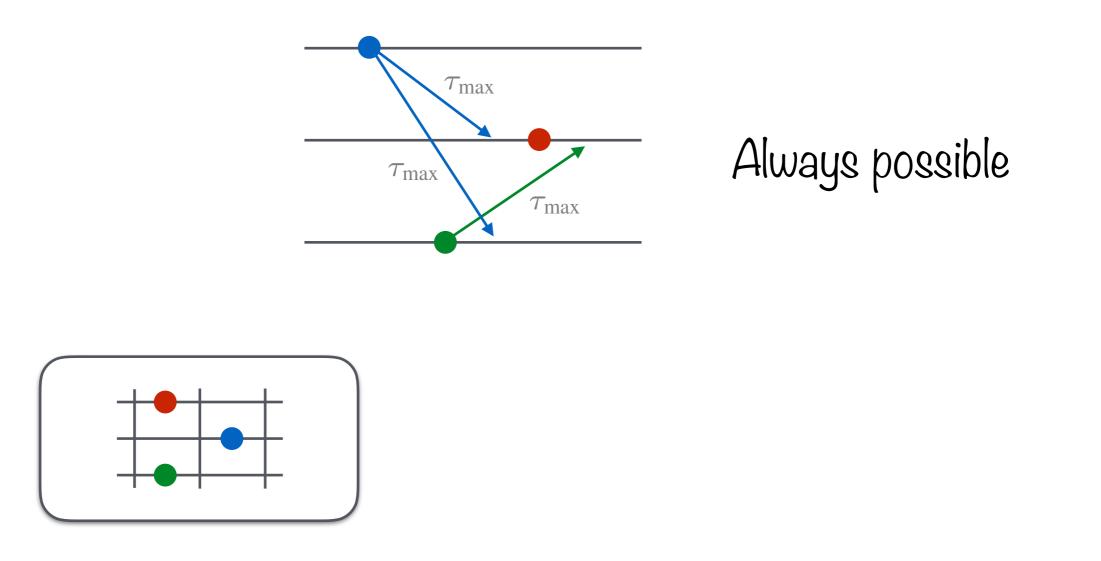
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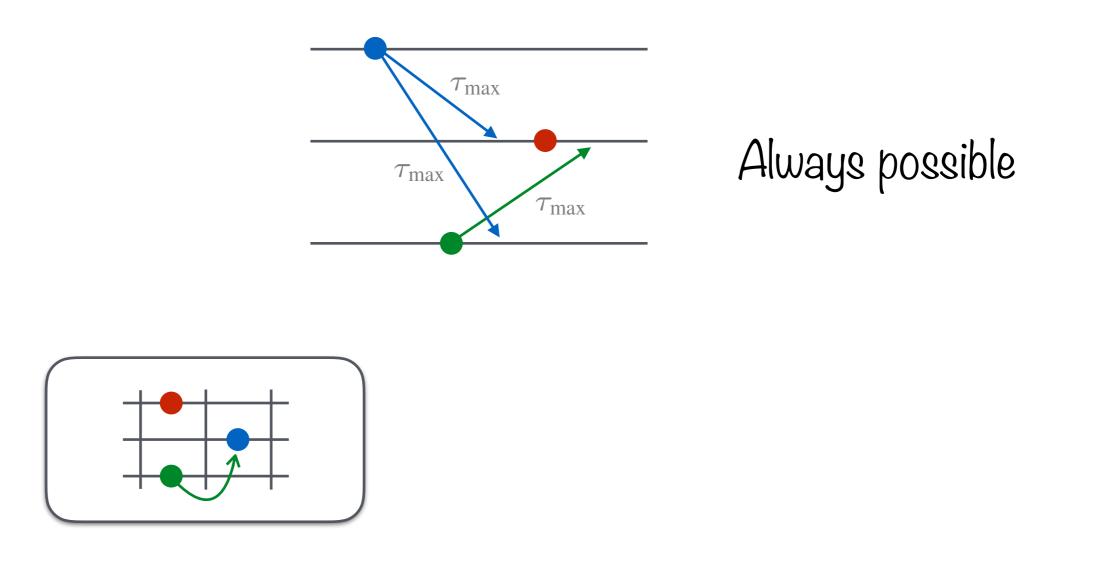


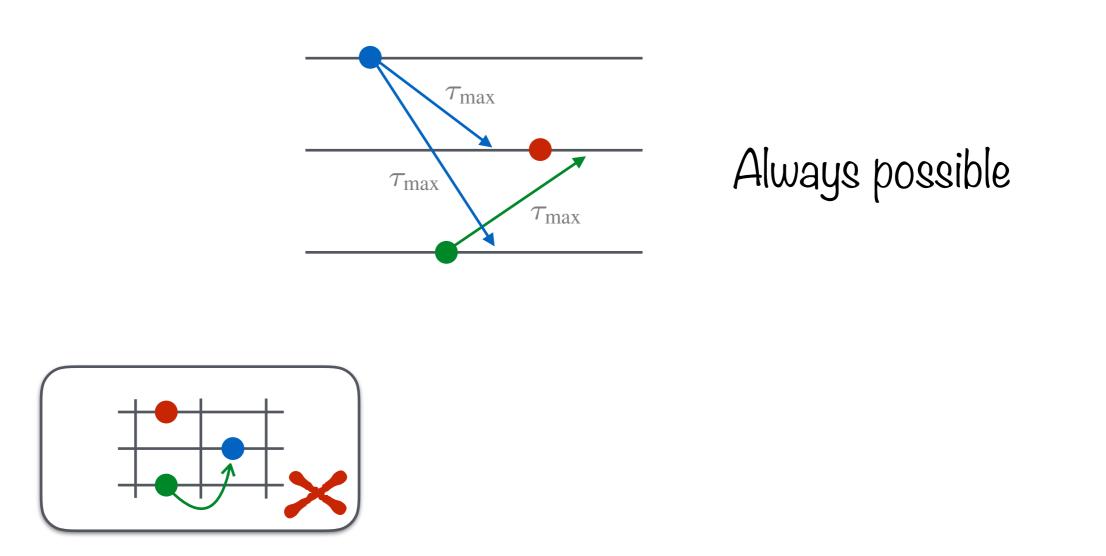
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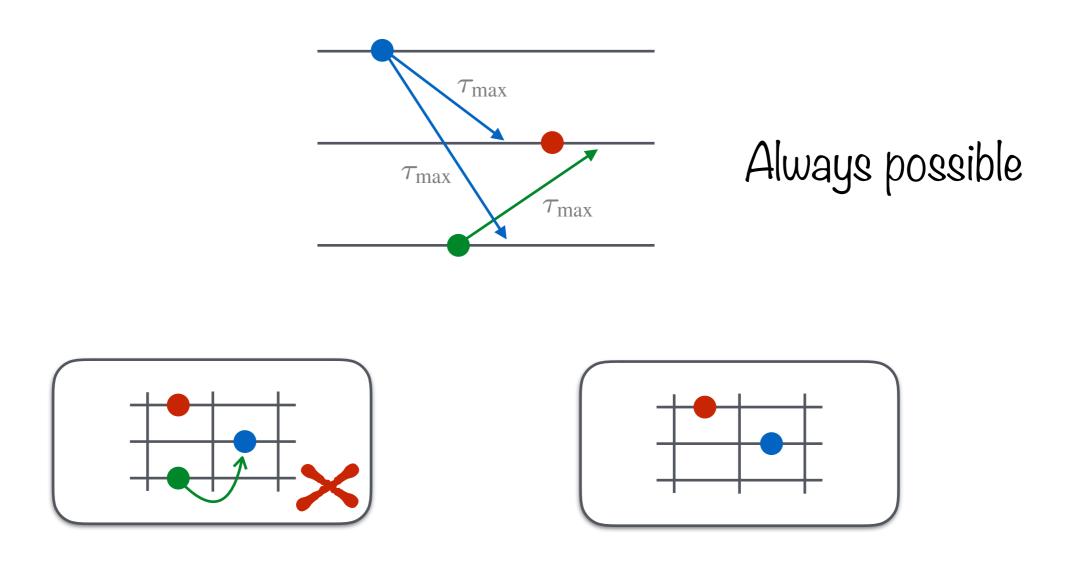


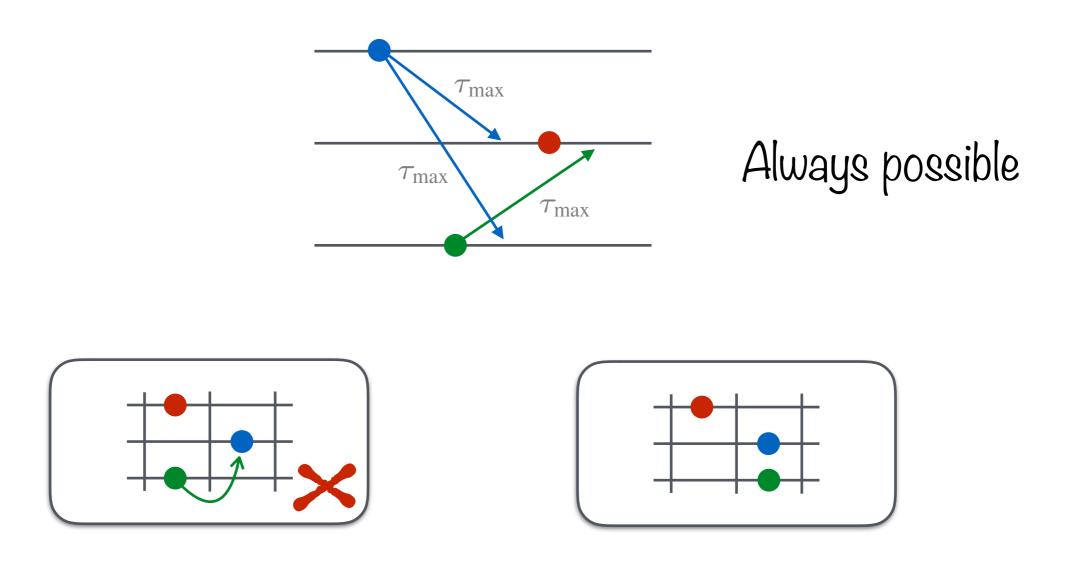
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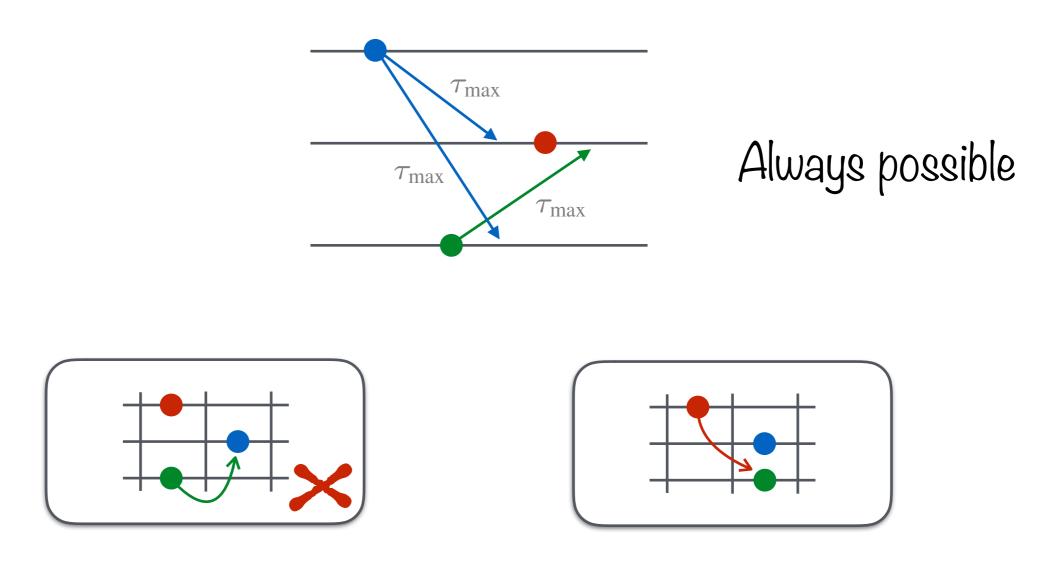


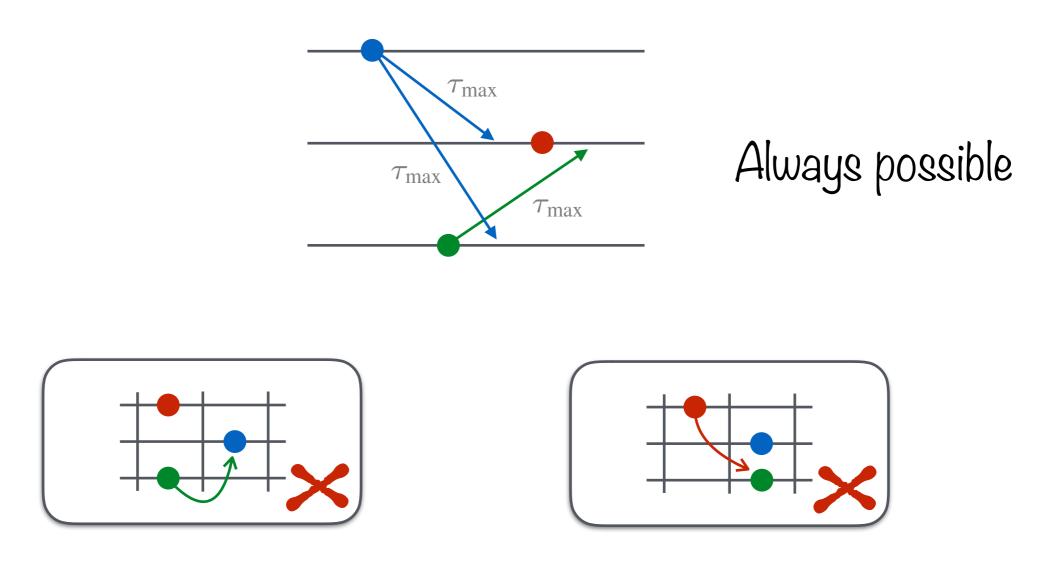


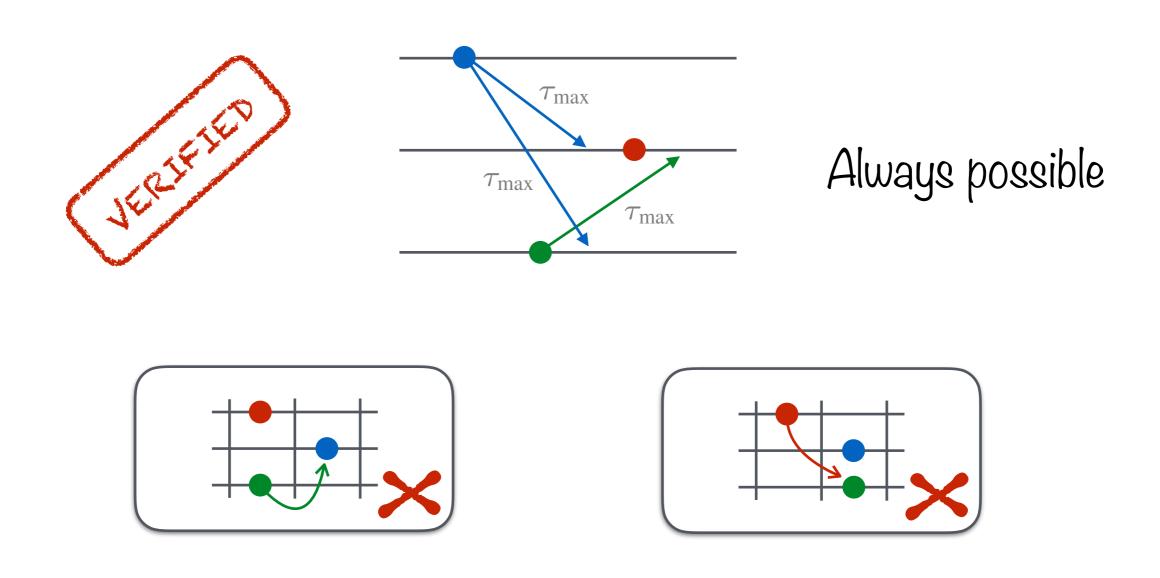












Proposition: If *f* is a unitary discretization for a trace, for a pair of nodes where $A \Rightarrow B$ we have that

 $A_i \to B_j \Longrightarrow f(A_i) < f(B_j),$ $A_i \not\to B_j \Longrightarrow f(A_i) \ge f(B_j).$

We gather all these constraints in a weighted graph

Vertices: Activations of the nodes **Edges:**

- If $A_i \to B_j$ then $A_i \xrightarrow{1} B_j$
- If $A \rightrightarrows B$ and $A_i \not\rightarrow B_j$ then $B_j \xrightarrow{0} A_i$

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Proof:

- If there is a cycle of positive weight, there is an event A_i such that $f(A_i) < f(A_i)$
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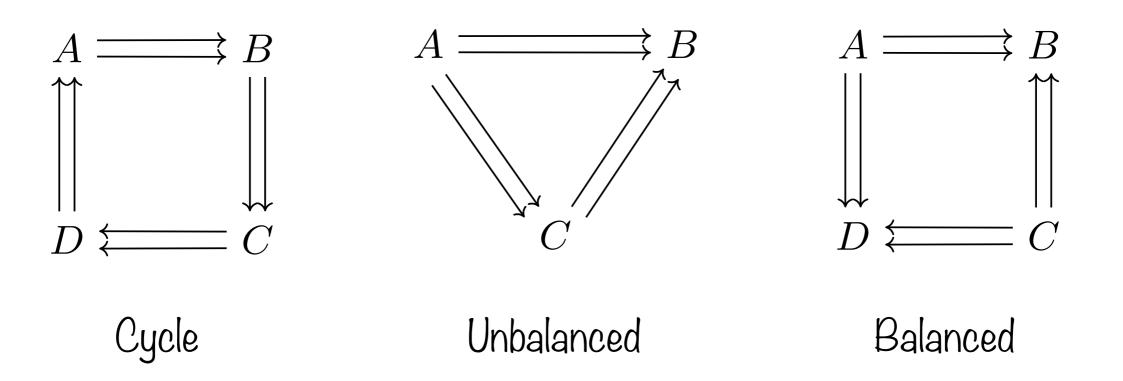
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Leave room for all the predecessors...

Proposition: A cycle of positive weight can be reduced to a cycle of positive weight based on a *u*-cycle of the communication graph.

u-cycle: cycle of the undirected communication graph



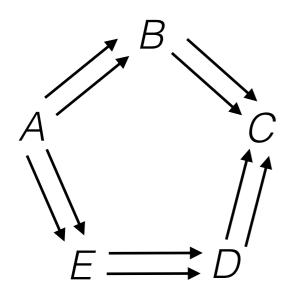
 L_c : size of the longest elementary communication cycle

Theorem: A quasi-periodic architecture is unitary discretizable if and only if

- 1. all *u*-cycle of the communication graph are either cycles or balanced *u*-cycle, and,
- 2. there is no balanced *u*-cycle in the communication graph or $\tau_{min} = \tau_{max}$, and,
- 3. there is no cycle in the communication graph, or

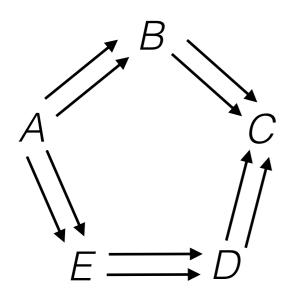
$$T_{min} \ge L_c \tau_{max}$$

Proof: If there is a *u*-cycle, construction of a counter-example

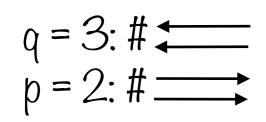




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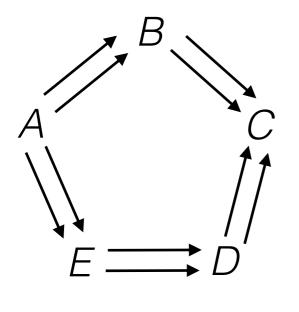




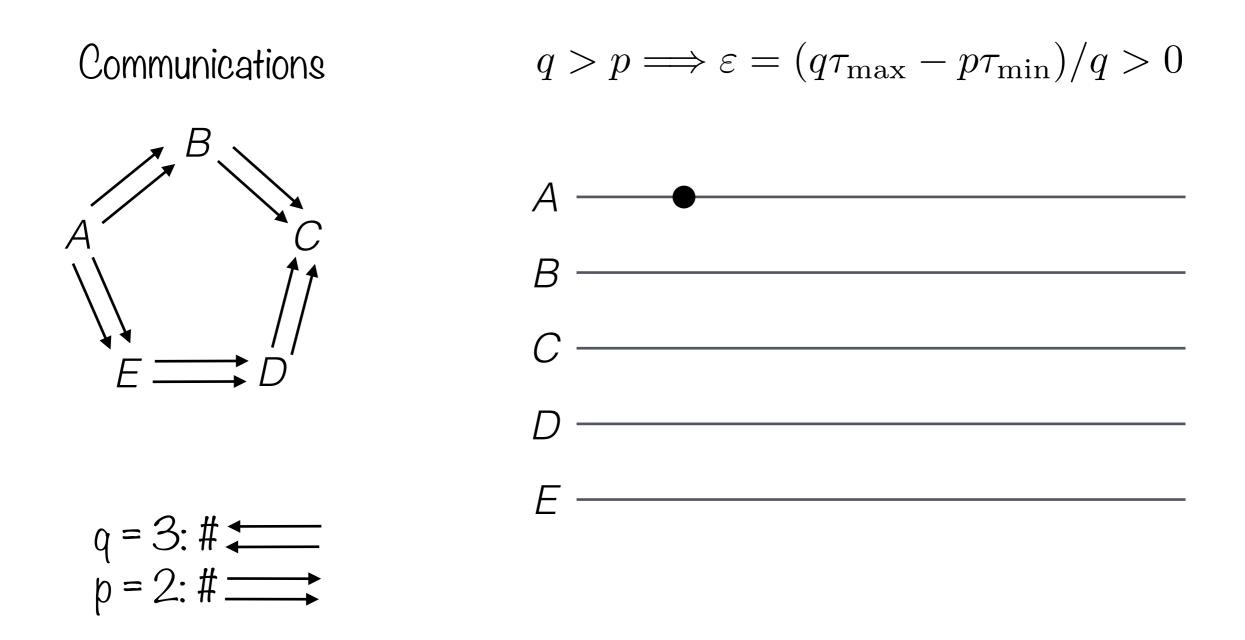


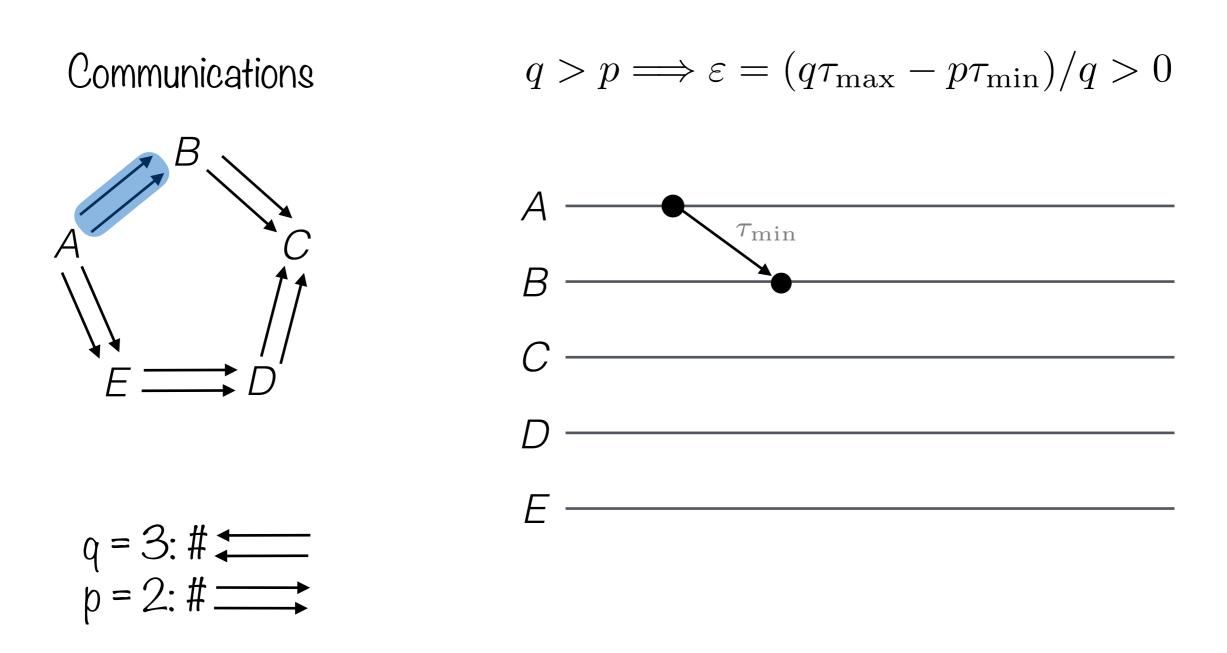
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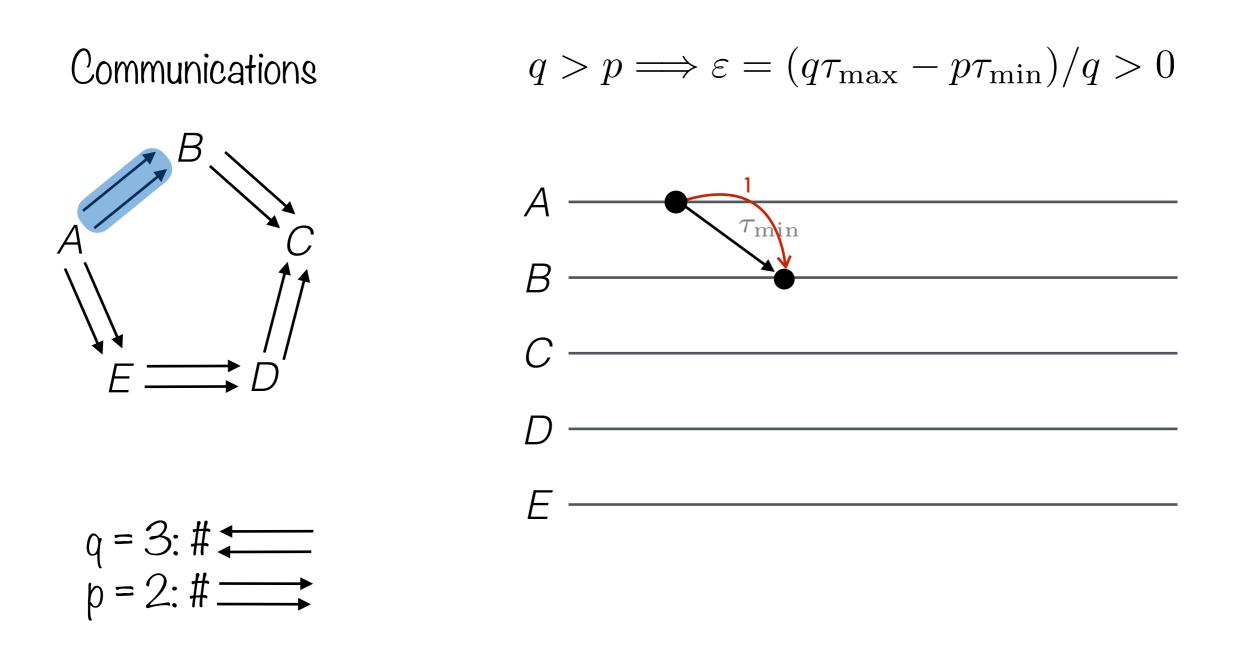
$$q > p \Longrightarrow \varepsilon = (q\tau_{\max} - p\tau_{\min})/q > 0$$

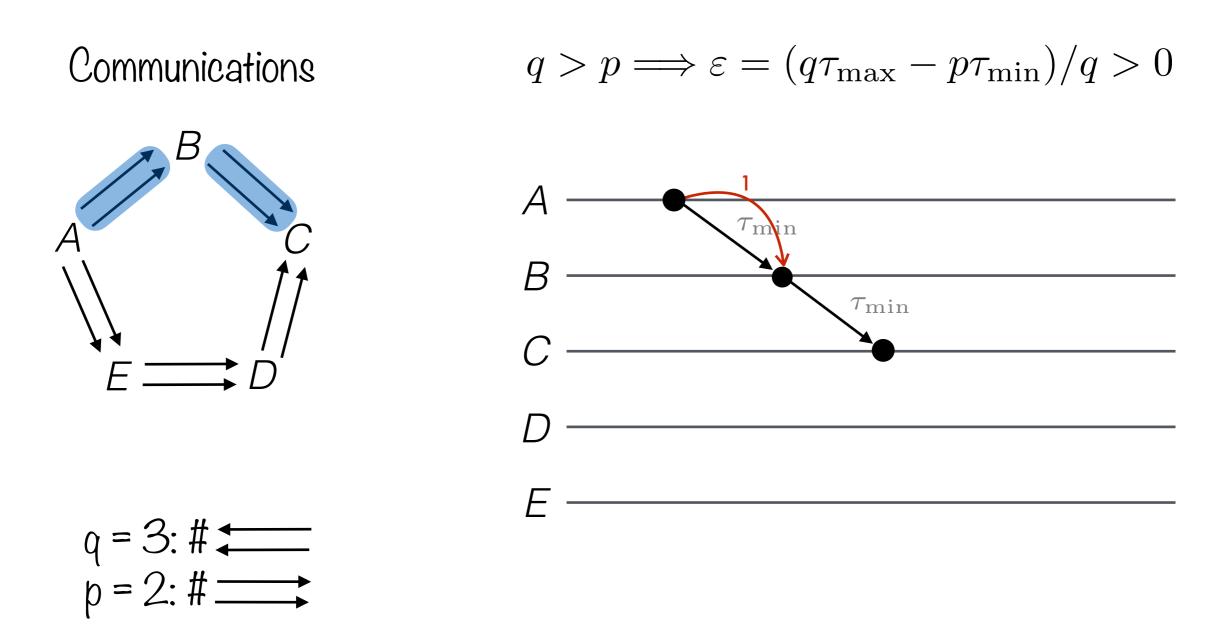


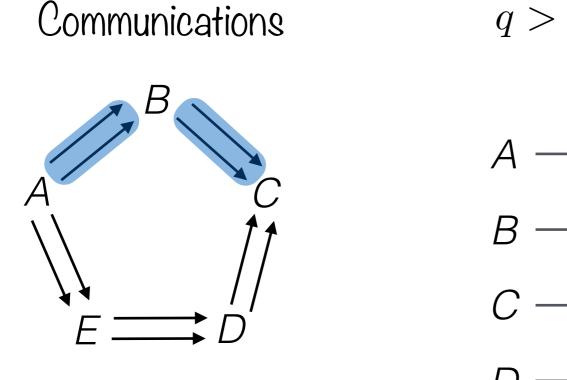




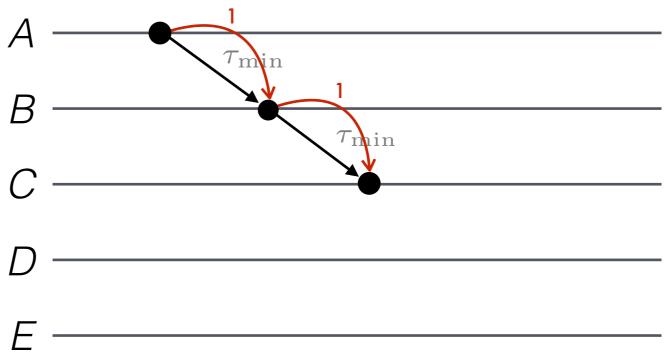






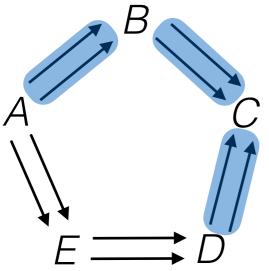


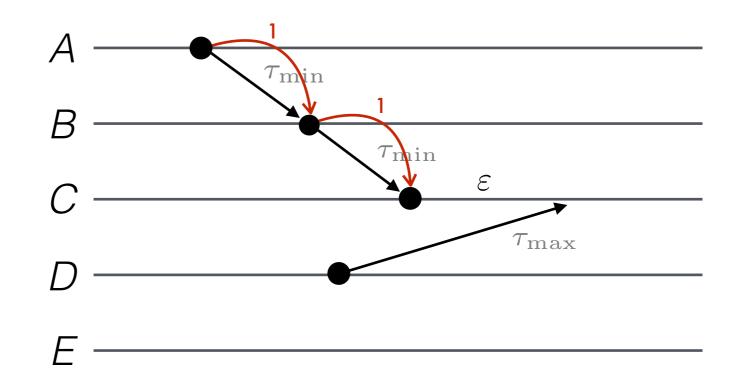
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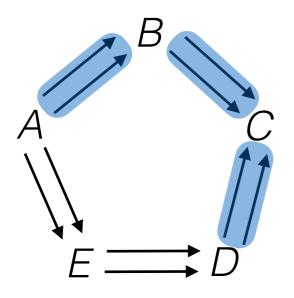
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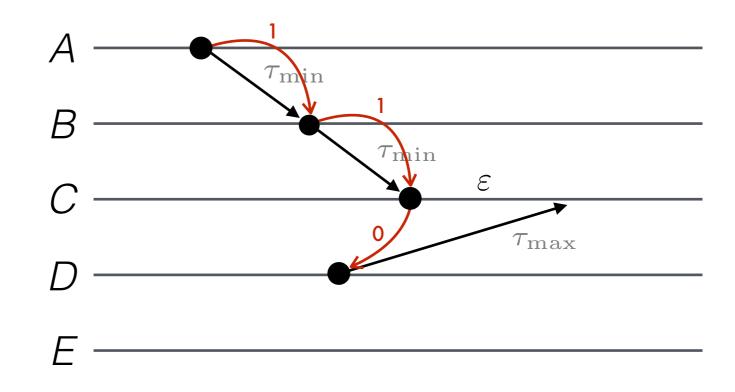




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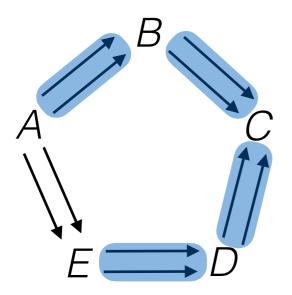
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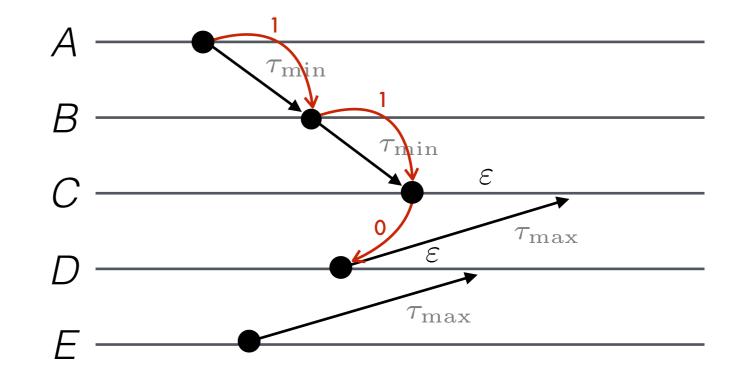




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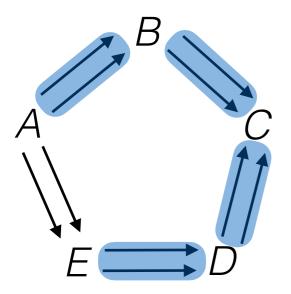
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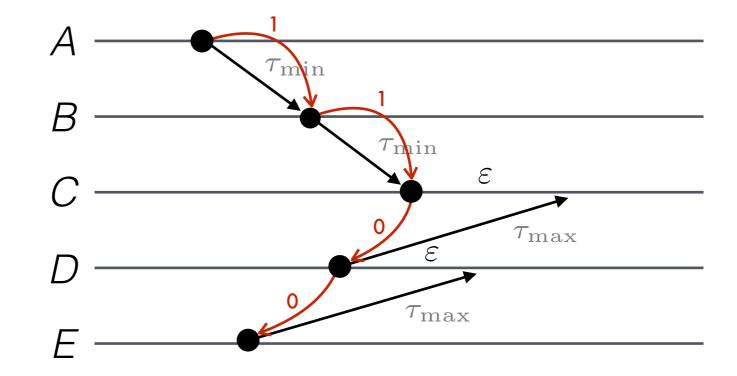




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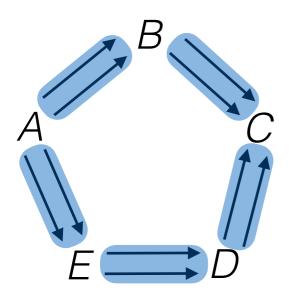
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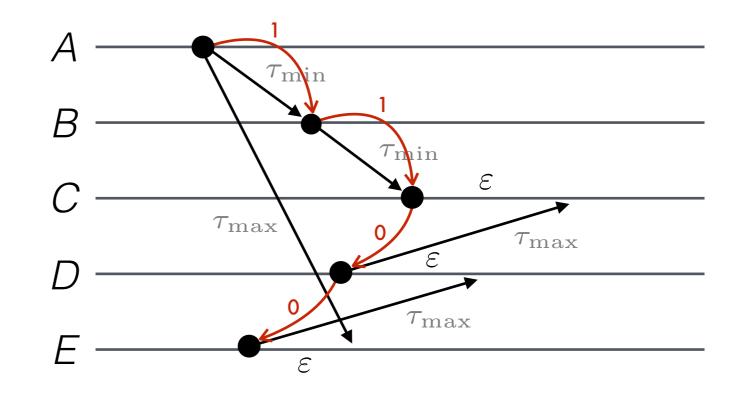




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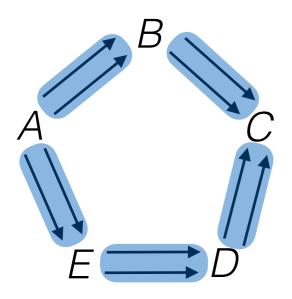
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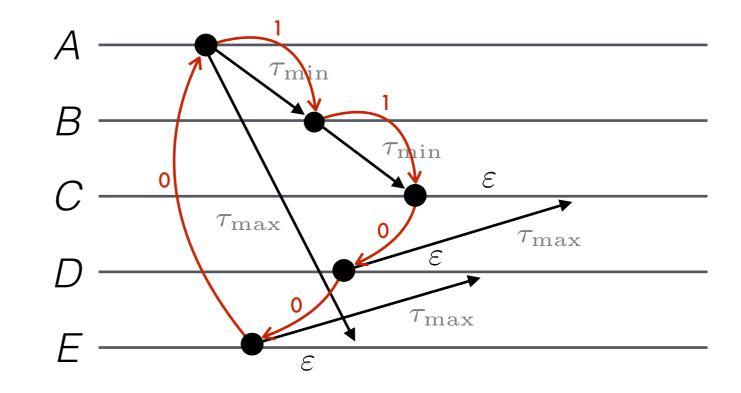




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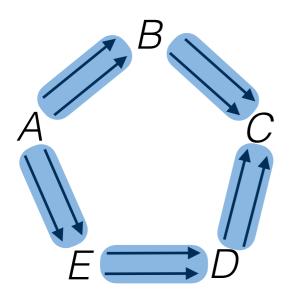
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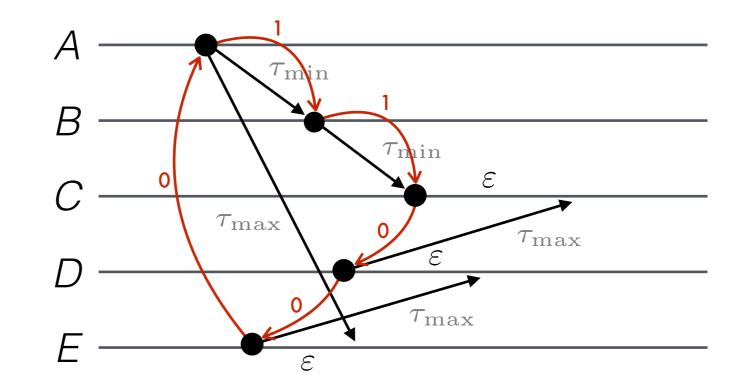




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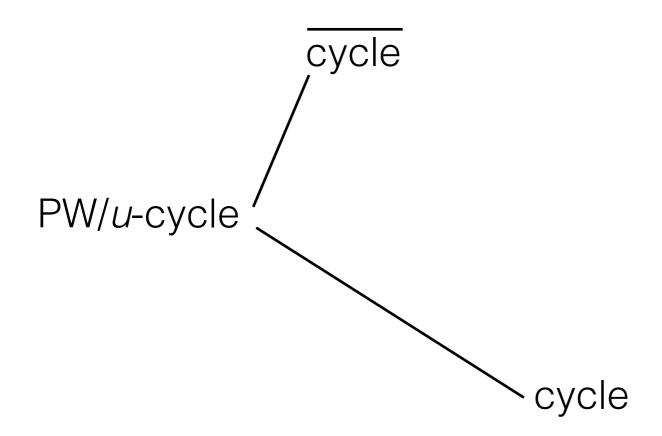


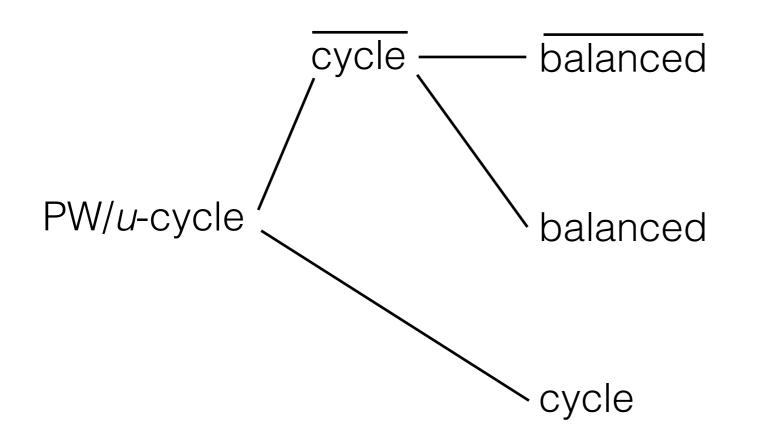
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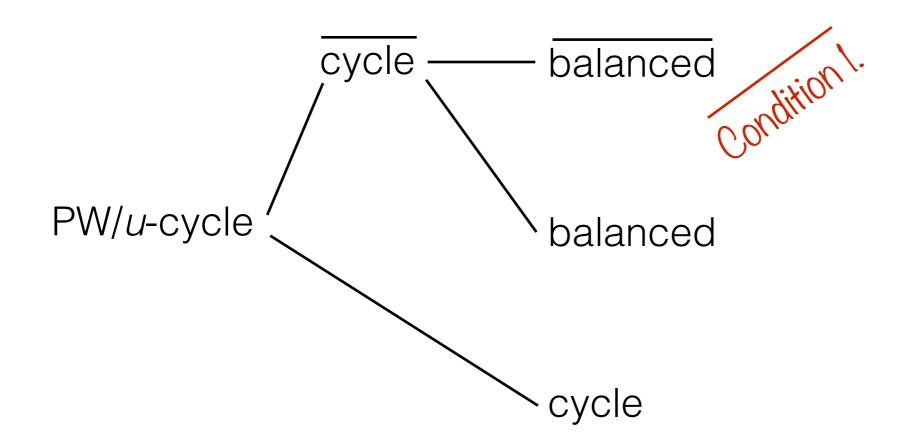
q = 3: # **↓** p = 2: # **↓**

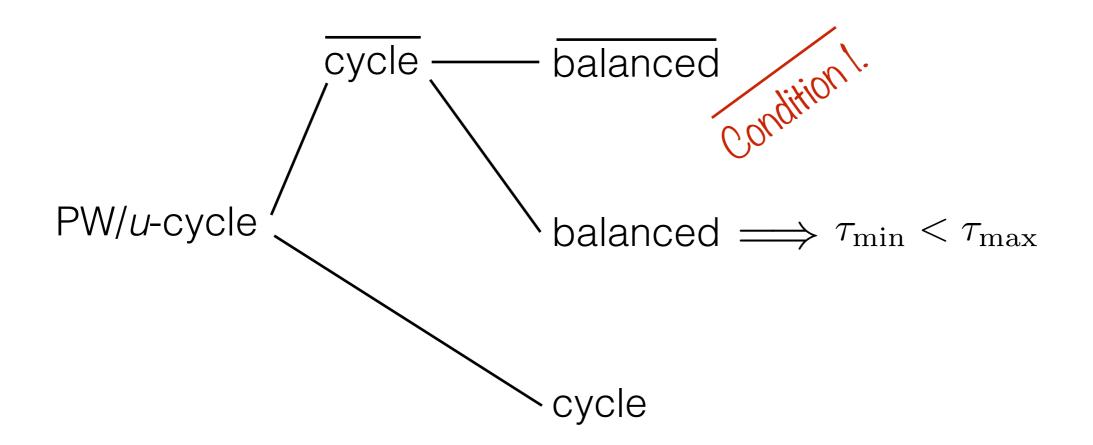
Proof: On the other hand, by contraposition,

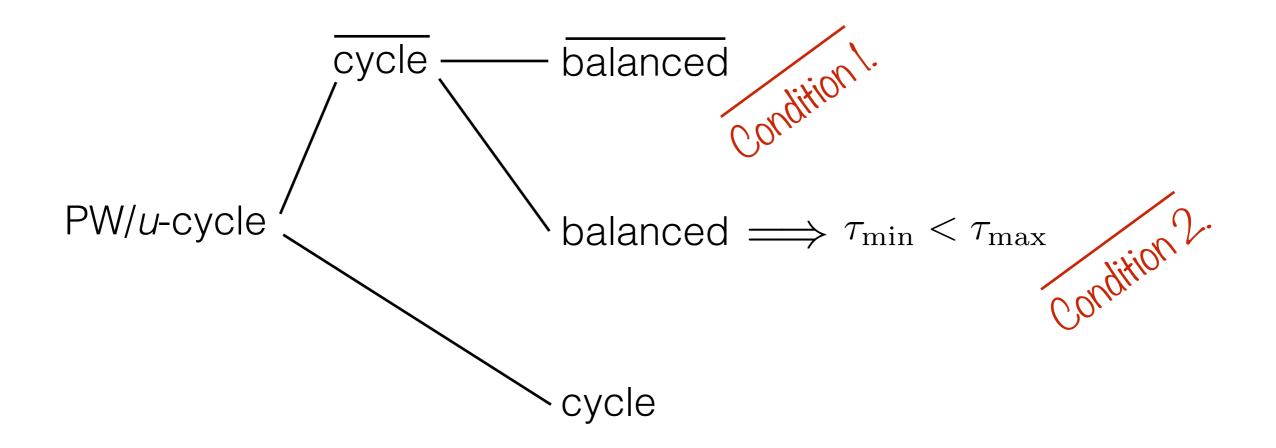
PW/u-cycle

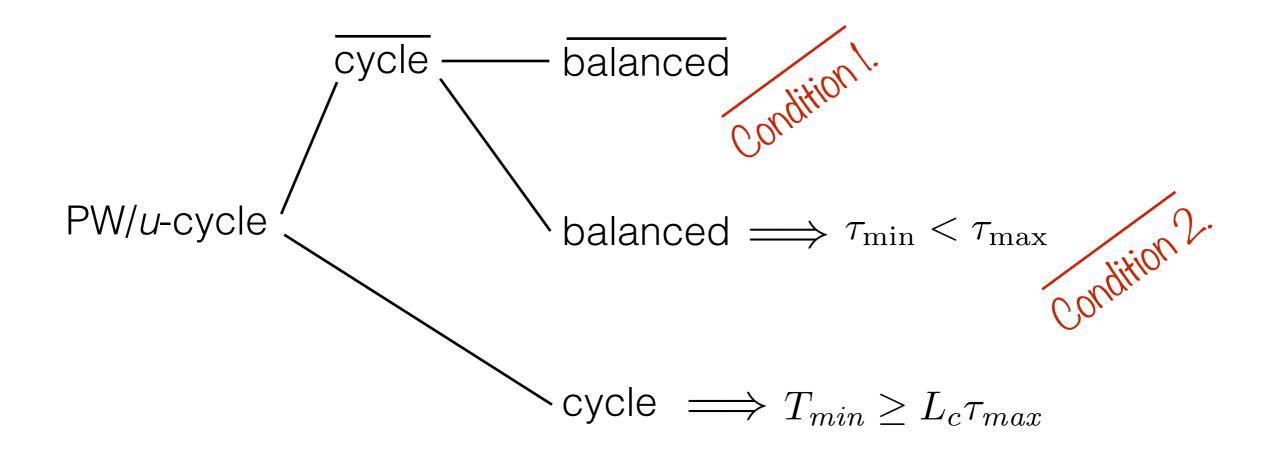






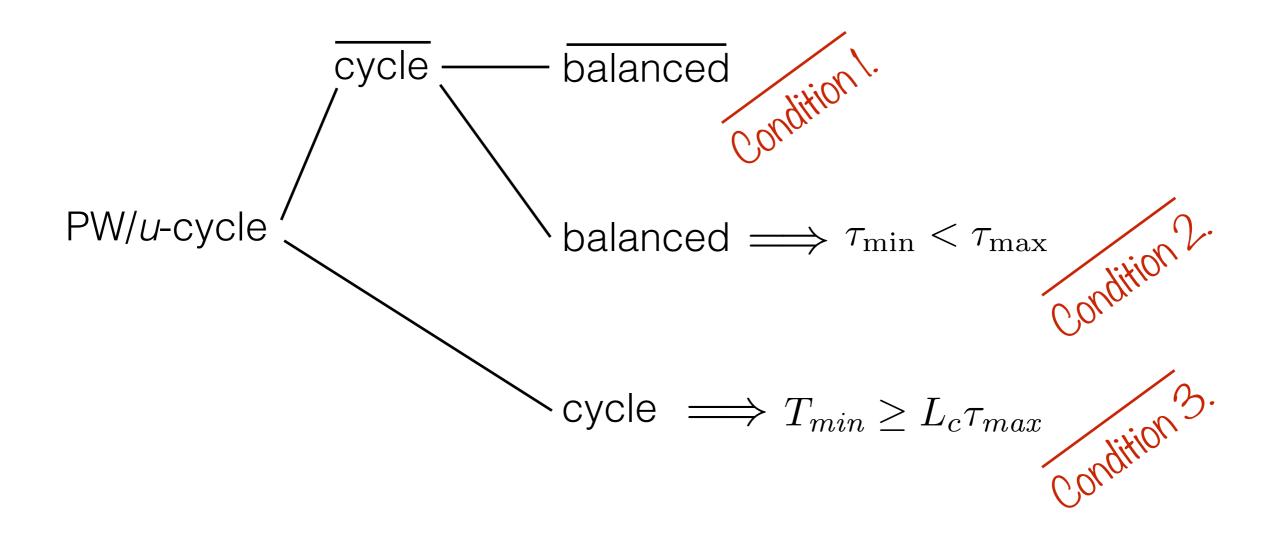






Constraining Communications

Proof: On the other hand, by contraposition,



Any of the two clocks cannot take the value "t" more than twice between two successive "t" values of the other one.

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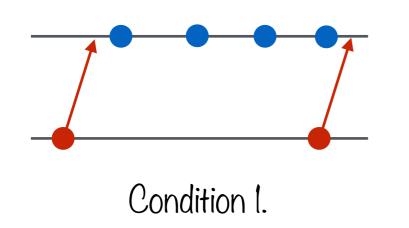
But there is no direct link between discrete- and real-time

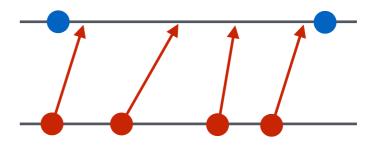
Any of the two clocks cannot take the value "t" more than twice between two successive "t" values of the other one.

But there is no direct link between discrete- and real-time

For any node:

there is no more than n activations between two message receptions
there is no more than n message receptions between two activations





Condition 2.

Any of the two clocks cannot take the value "t" more than twice between two successive "t" values of the other one.

But there is no direct link between discrete- and real-time

Definition (*n***-Quasi-Synchrony):** A quasi-periodic architecture is *n*-quasi-synchronous if for every trace t

1. there exists a unitary discretization f, and

2. for any node $A \rightleftharpoons B$, there is no chain of activation of length greater than n, that is no i and j such that

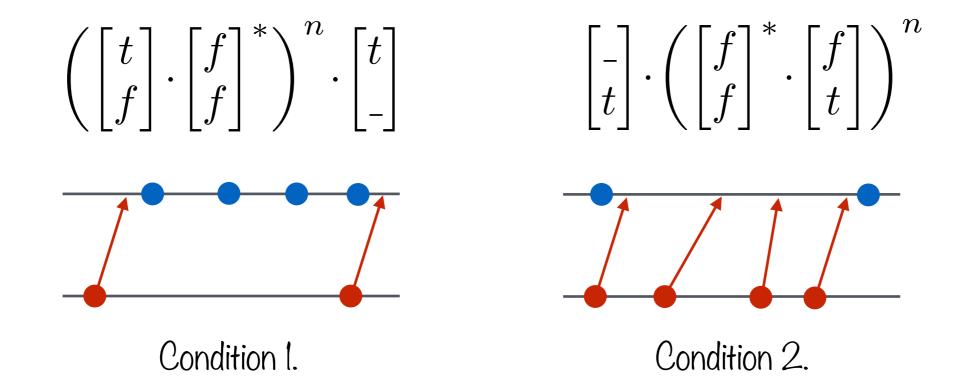
$$f(B_j) < f(A_i) < \dots < f(A_{i+n}) \le f(B_{j+1})$$

 $f(A_j) \le f(B_i) < \dots < f(B_{i+n}) < f(A_{j+1})$

This can be formalized by saying that the boolean vector stream composed of the two clocks should never contain the subsequence:

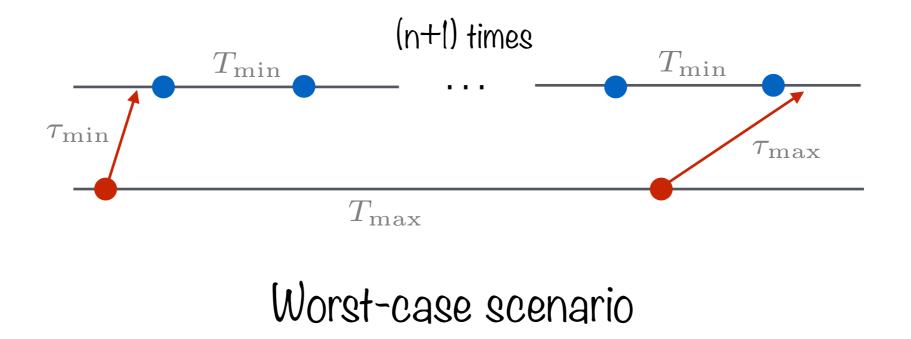
$$\left[\begin{array}{c}t\\-\end{array}\right]\cdot\left[\begin{array}{c}f\\f\end{array}\right]^*\cdot\left[\begin{array}{c}t\\f\end{array}\right]\cdot\left[\begin{array}{c}f\\f\end{array}\right]^*\cdot\left[\begin{array}{c}t\\-\end{array}\right]$$

The boolean vector associated to nodes A and B never contains either of the subsequences



Theorem: A quasi-periodic architecture is *n*-quasi-synchronous if and only if

- 1. the conditions for unitary discretizability hold, and,
- 2. $nT_{min} + \tau_{min} \geq T_{max} + \tau_{max}$.



The quasi-synchronous abstraction is a nice idea to reduce possible interleavings when using verification tools for discrete models.

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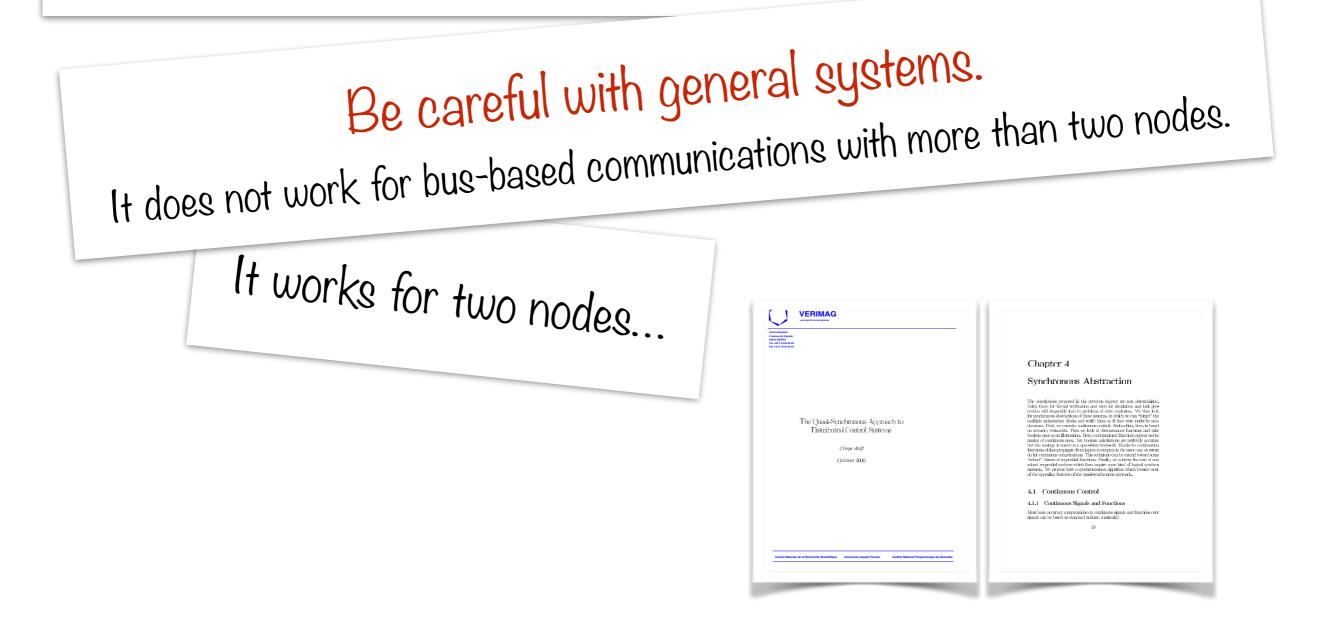
Be careful with general systems. It does not work for bus-based communications with more than two nodes.

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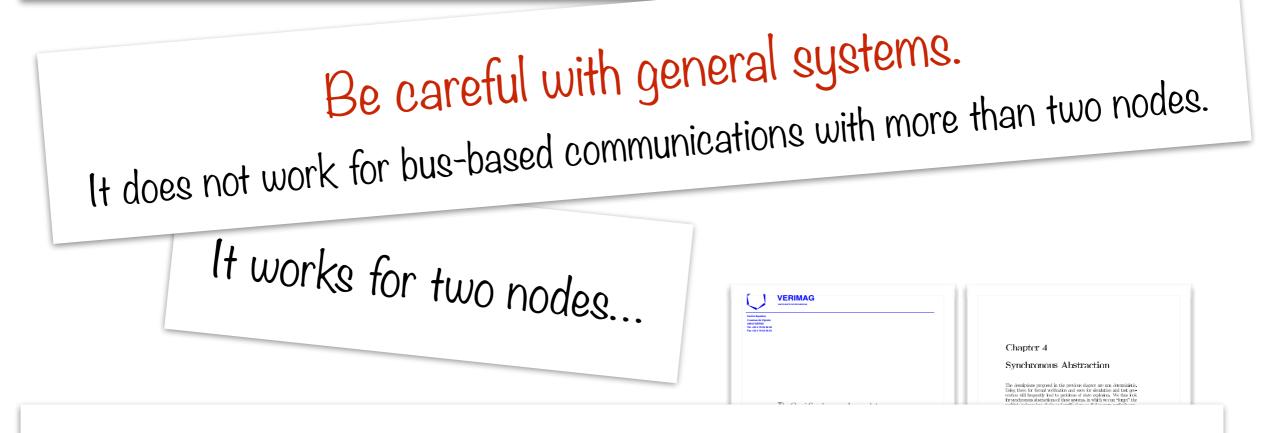
Be careful with general systems. It does not work for bus-based communications with more than two nodes.

It works for two nodes...

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The descriptions proposed in the previous chapter are non deterministic. Using them for formal verification and even for simulation and test generation will frequently lead to problems of state explosion.