Symbolic Simulation of Clocks

Guillaume Baudart Timothy Bourke Marc Pouzet

Work In Progress

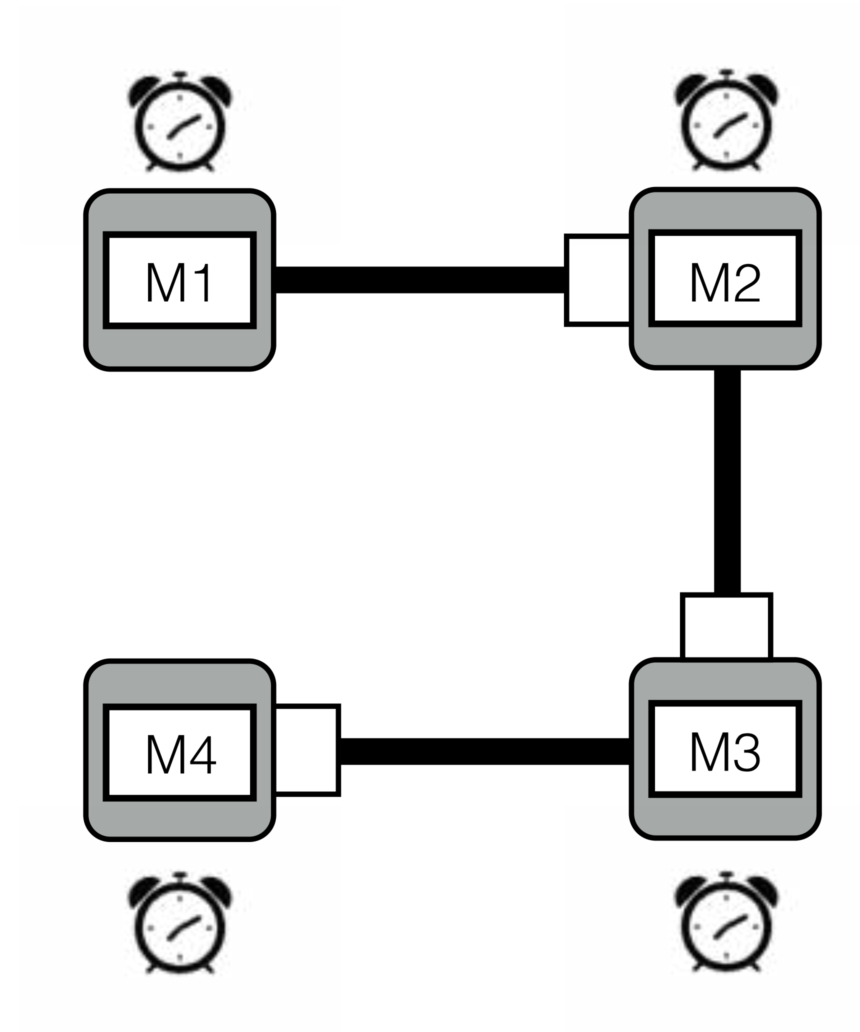
Example: Quasi-Periodic Architectures

- A set of “quasi-periodic” processes with local clocks and nominal period T^n (jitter ε)

$$0 < T_{\min} \leq T^n \leq T_{\max} \quad \text{or}$$
$$T^n - \varepsilon \leq \kappa_i - \kappa_{i-1} \leq T^n + \varepsilon$$

$(\kappa_i)_{i \in \mathbb{N}}$ clock activations

- Buffered communication without message inversion or loss



“Cooking Book” [Caspi 2000]

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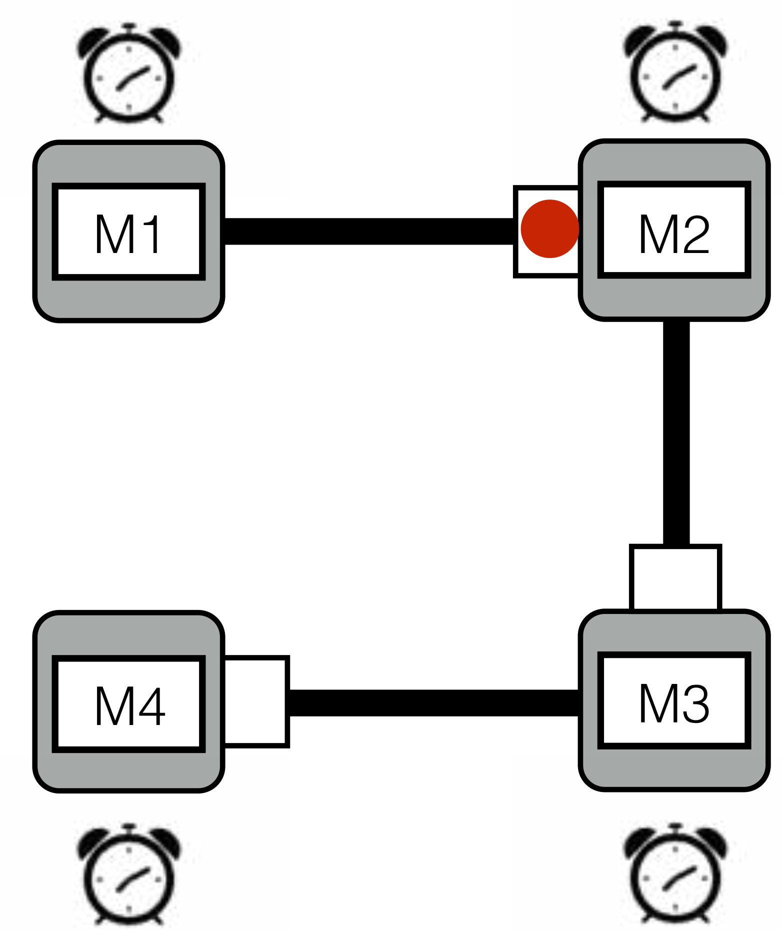

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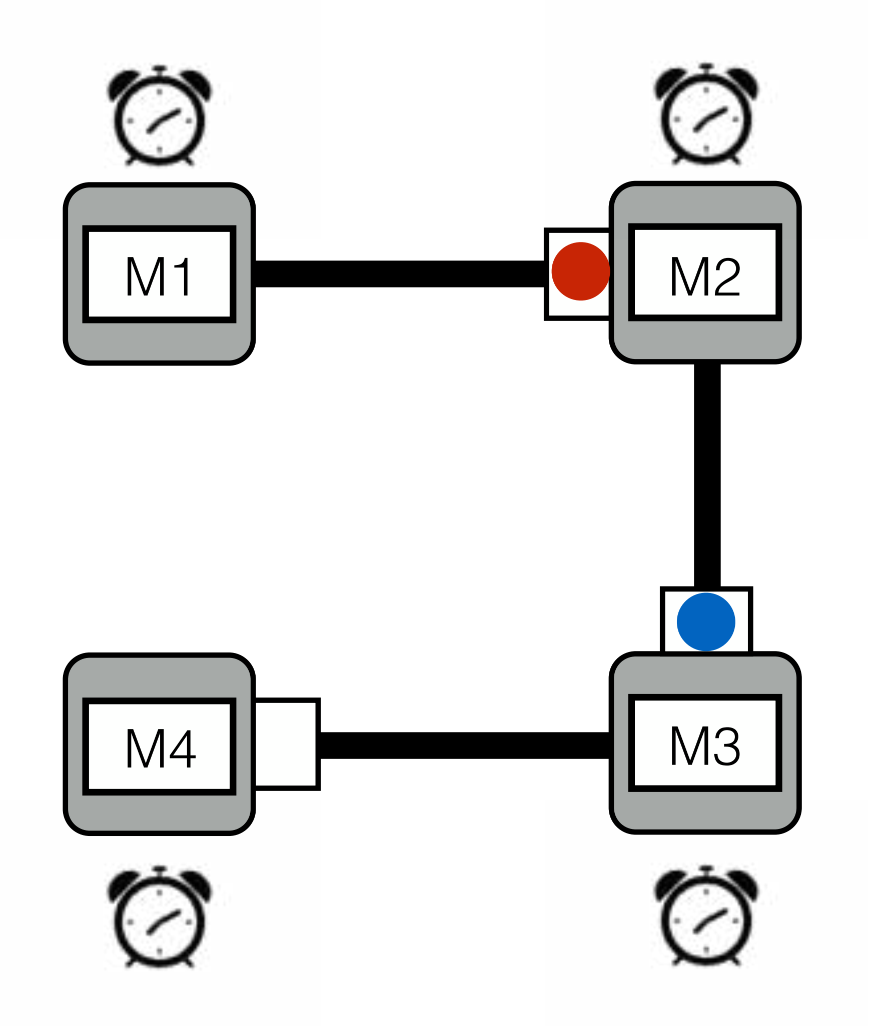

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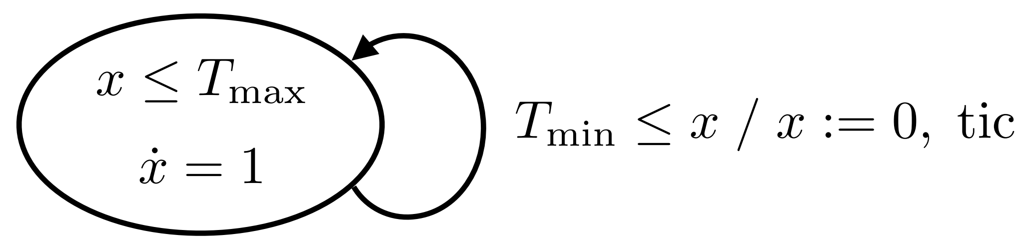
Example: Quasi-Periodic Architectures

Fuzzy Metronome



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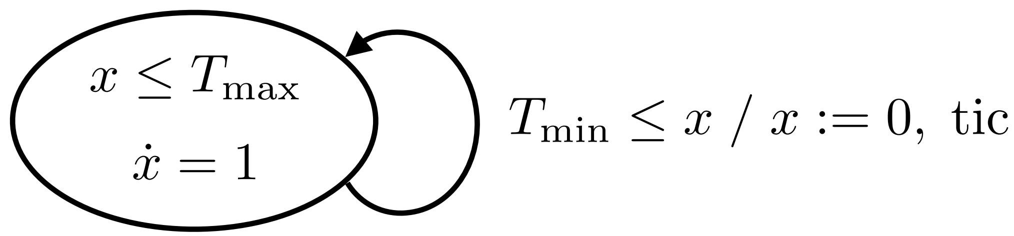


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  rec clock x reset tic()
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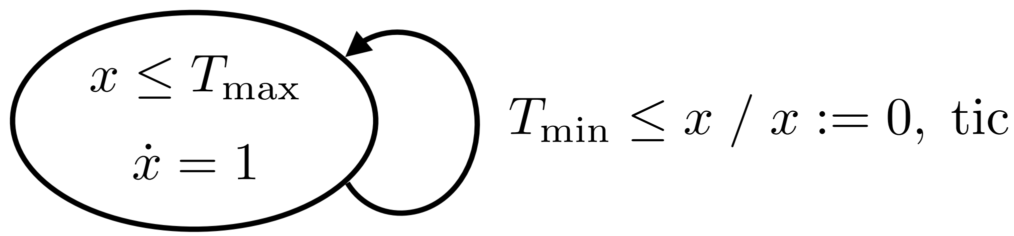


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Clock can be implemented as a simple ODE

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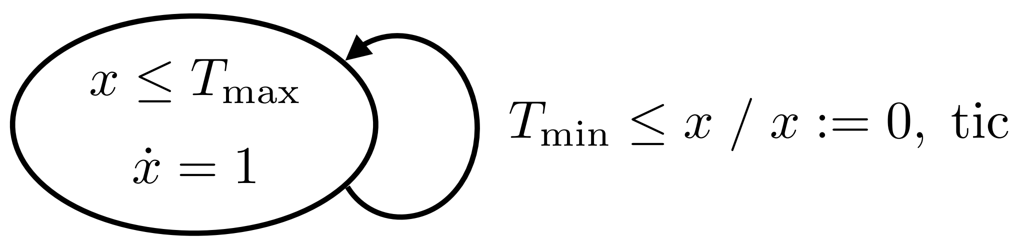


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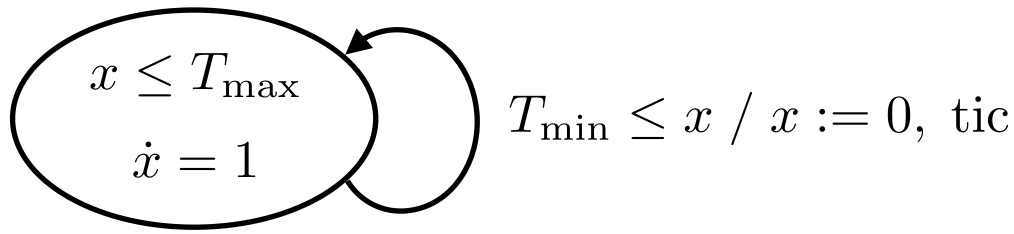

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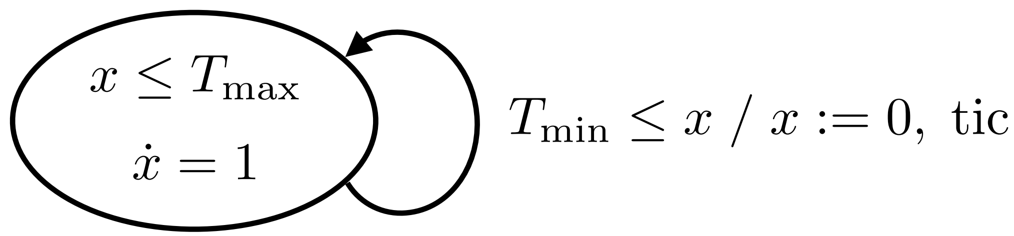

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```
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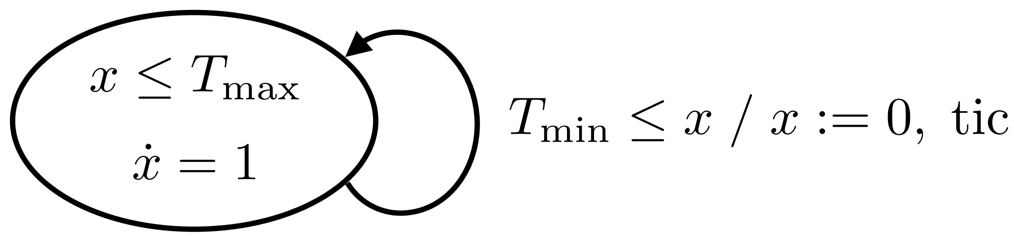

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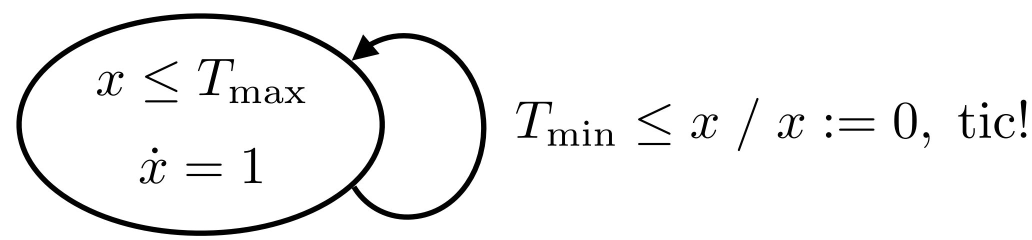

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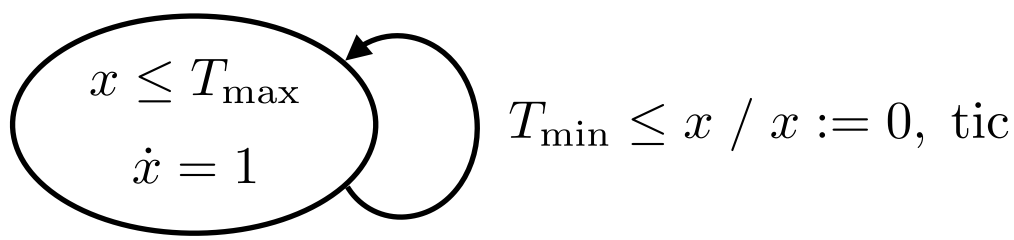

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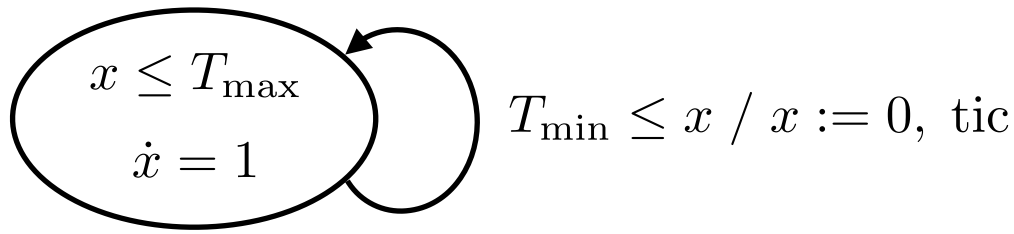

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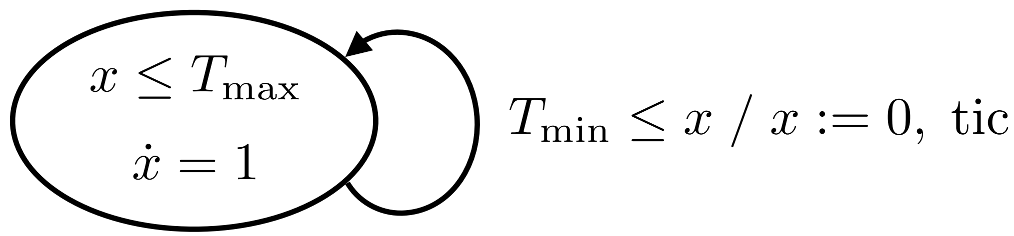

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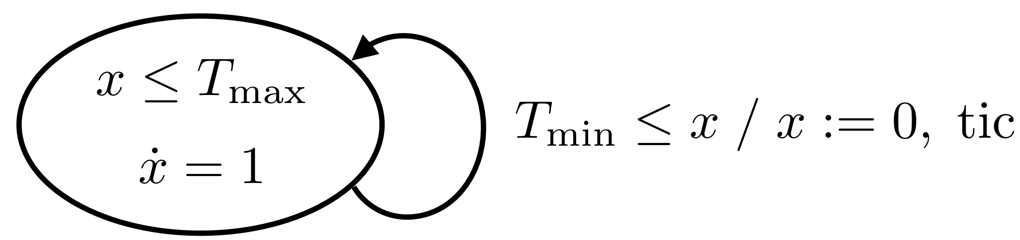

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$\texttt{[x = 0]} \longrightarrow \texttt{[x = 1.1]}$

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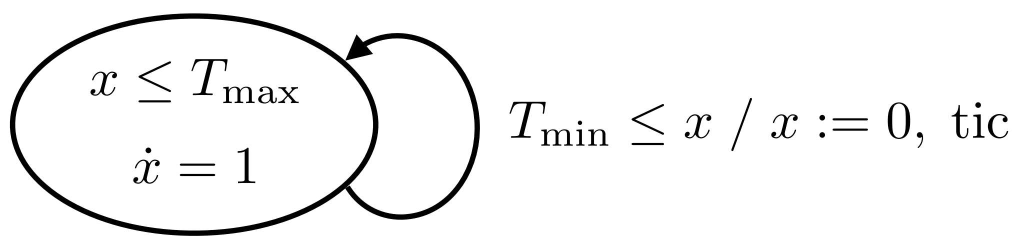

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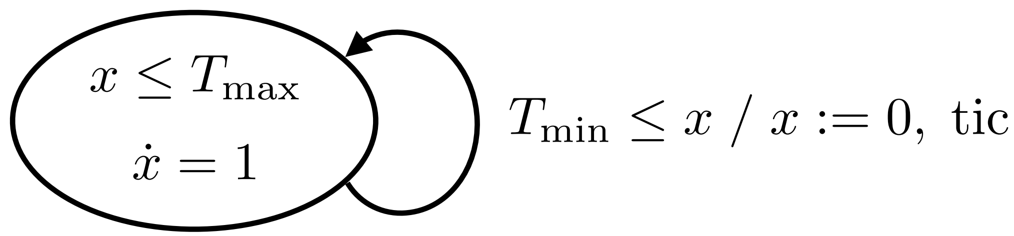

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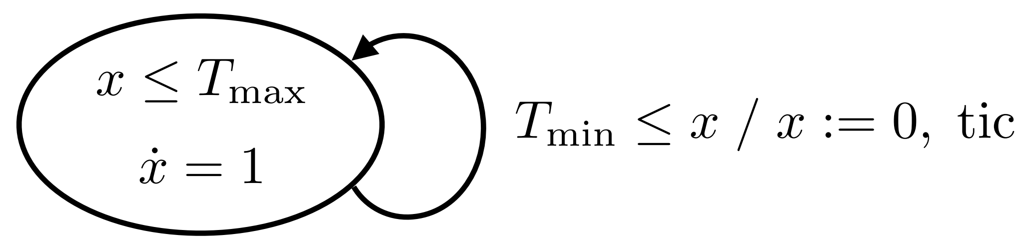

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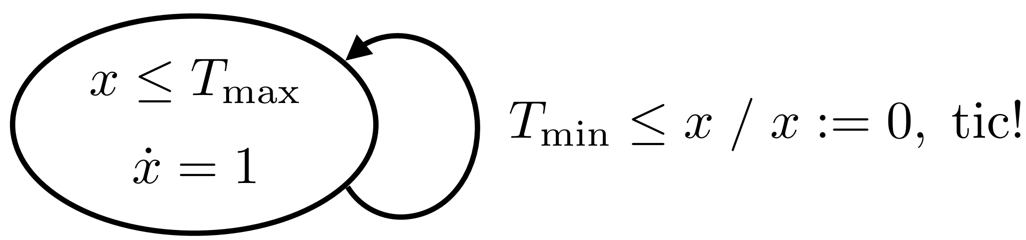

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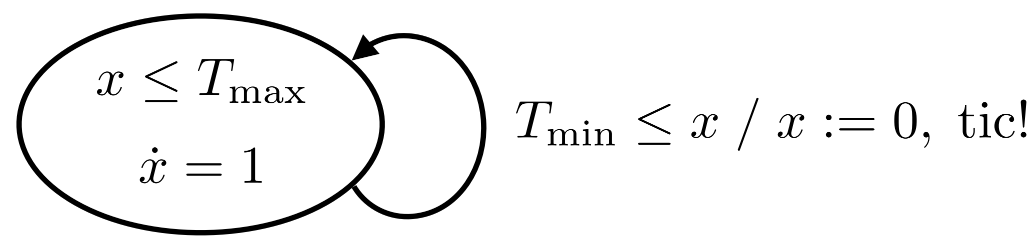

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[0≤x≤3]

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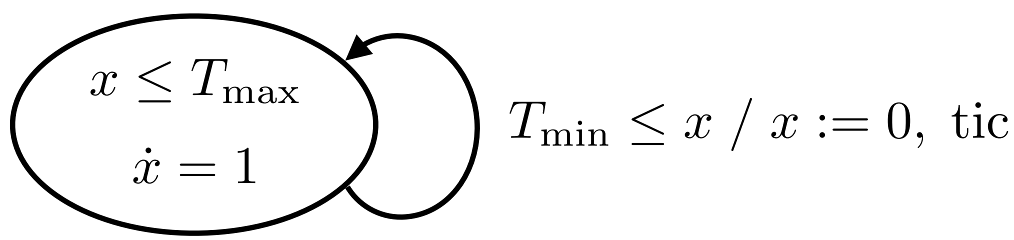

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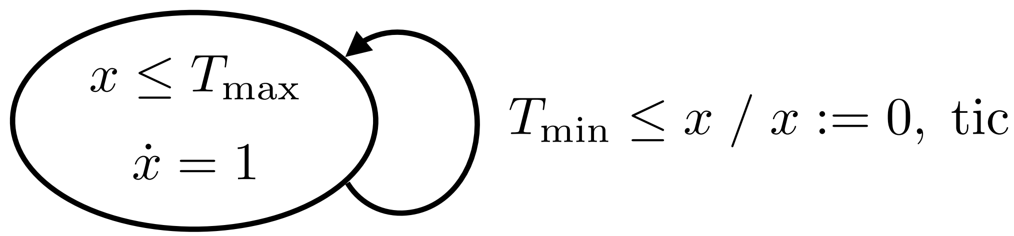

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$[0 \leq x \leq 3] \xrightarrow{\texttt{in\_x/}} [0 \leq x \leq 3] \xrightarrow{\texttt{wait}} [3 \leq x \leq 5]$

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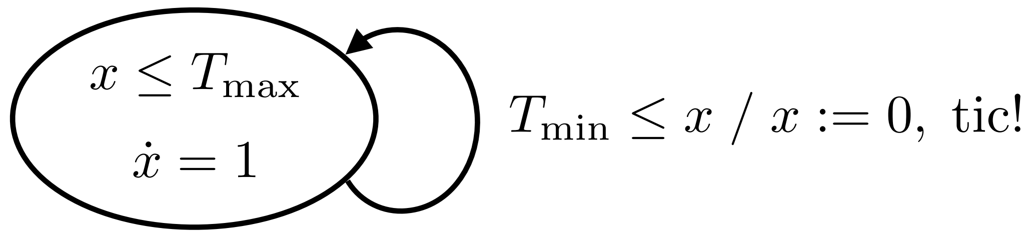

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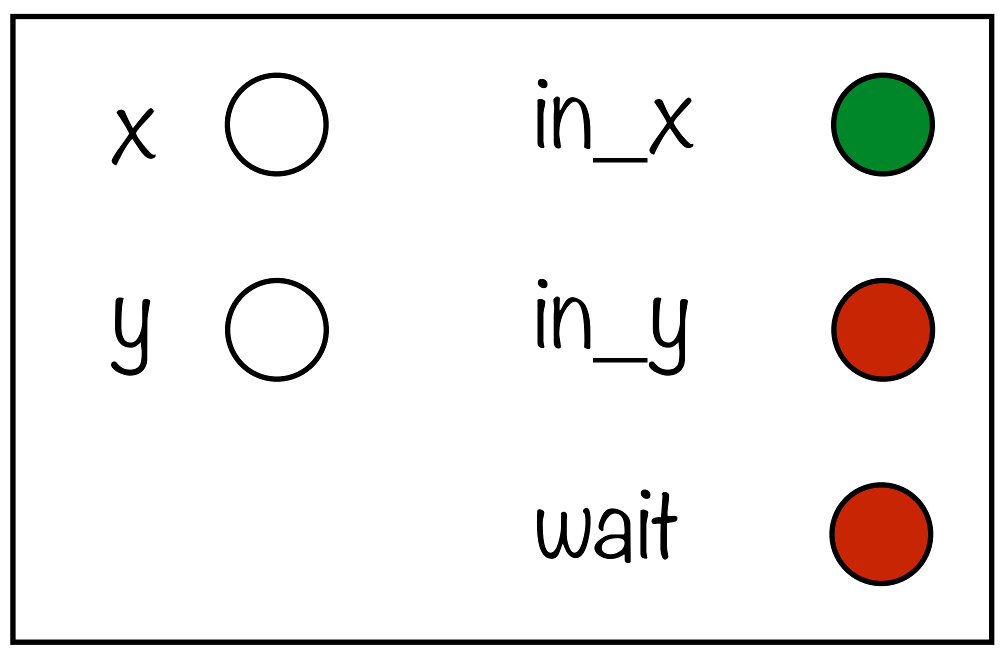

Simulation Box

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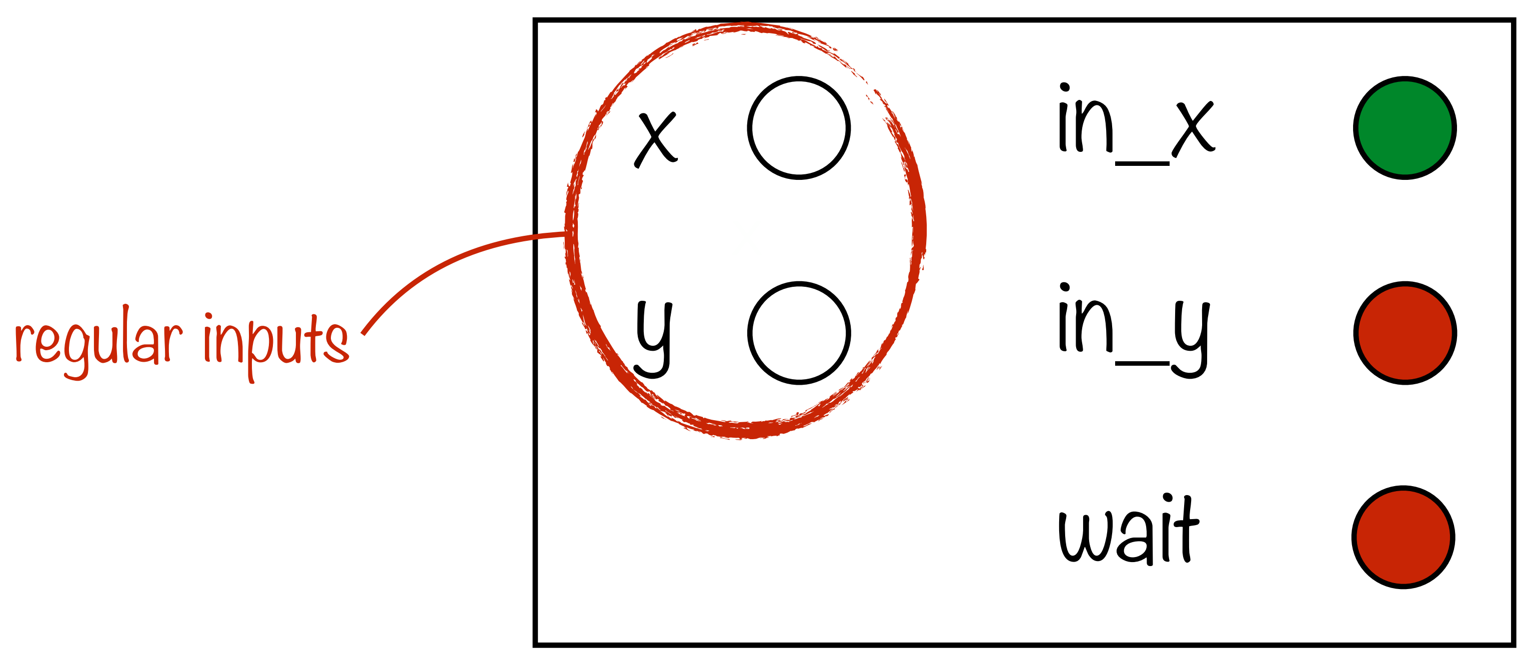

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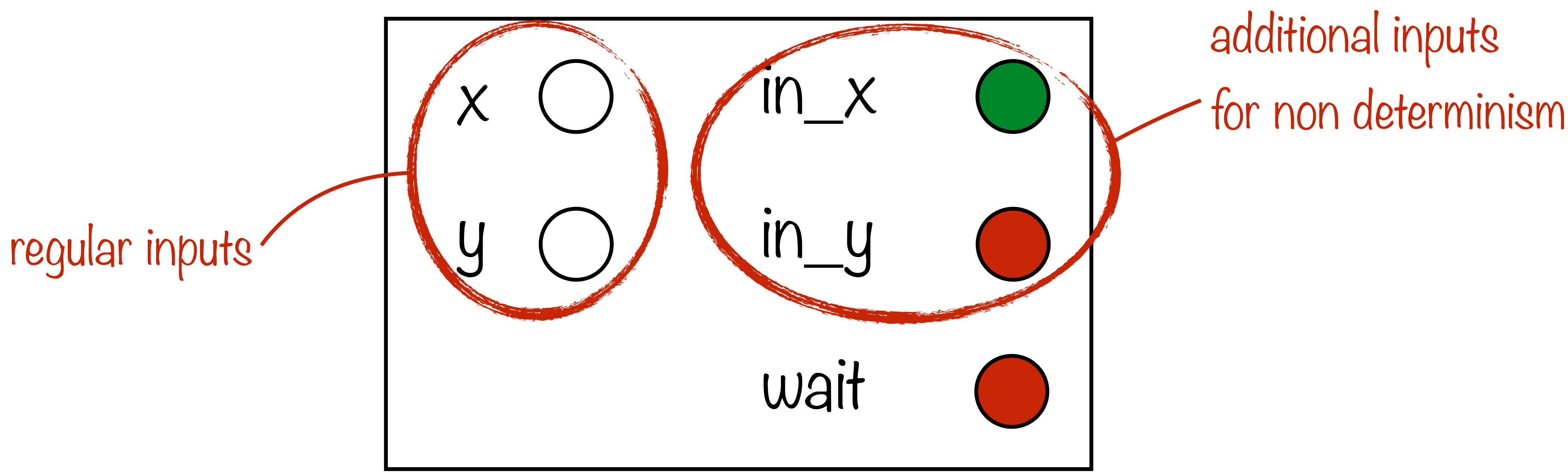

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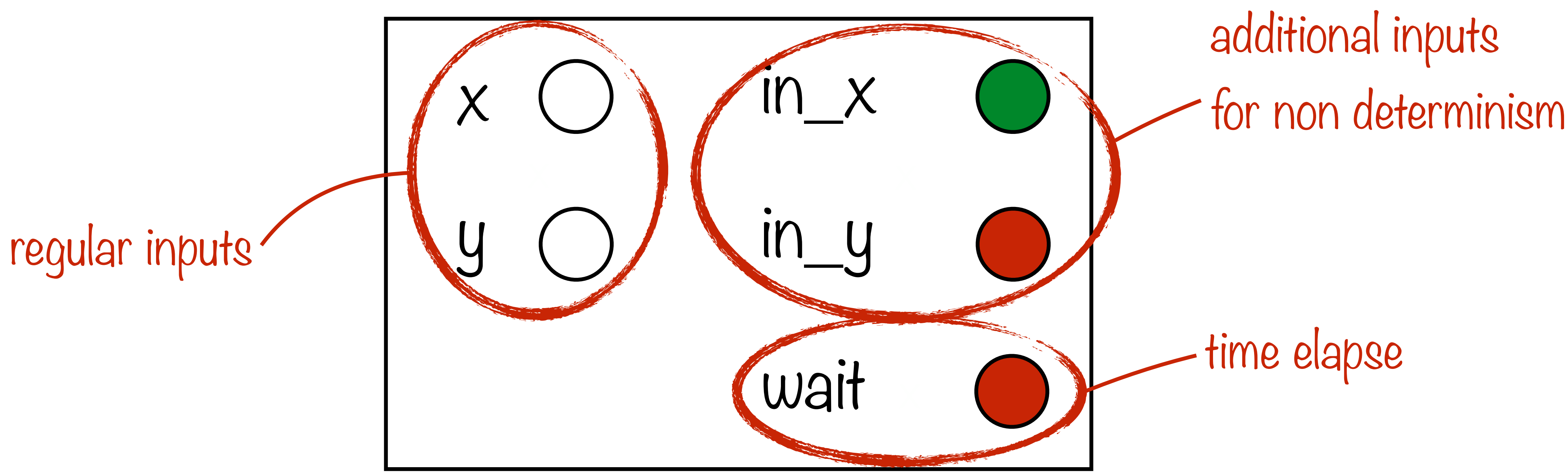

Simulation Box

Symbolic Simulation

Comparison with existing tool

Australian Walk

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Safe

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Comparison with existing tool

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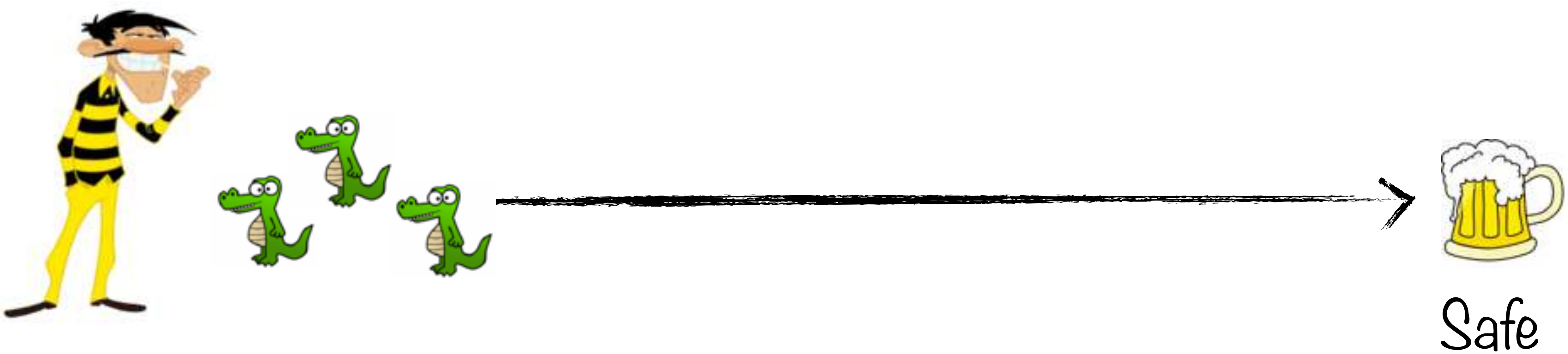

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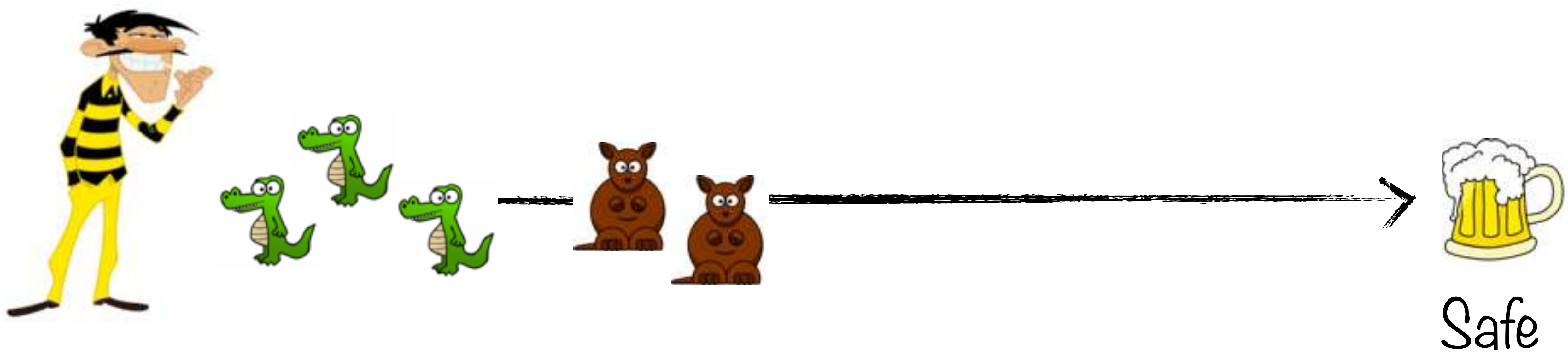

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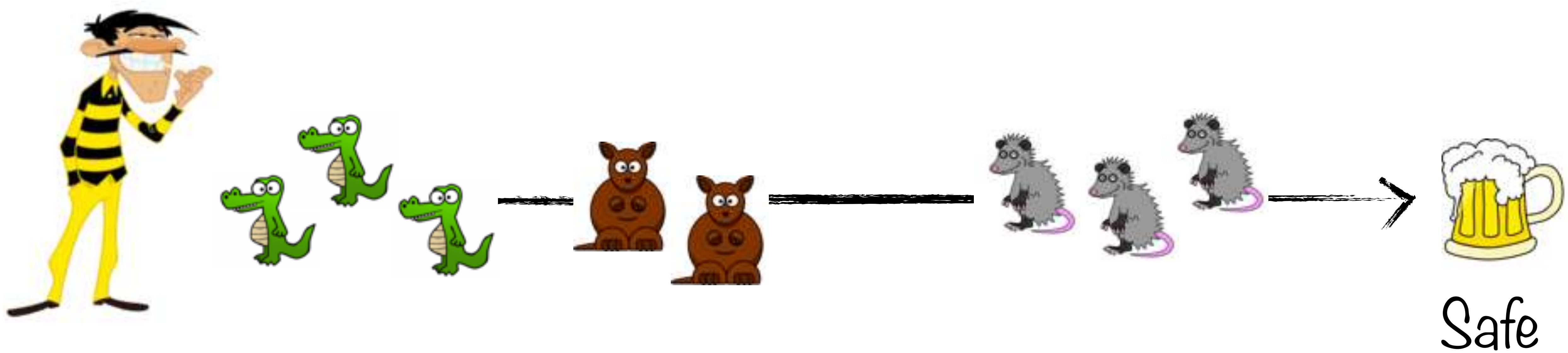

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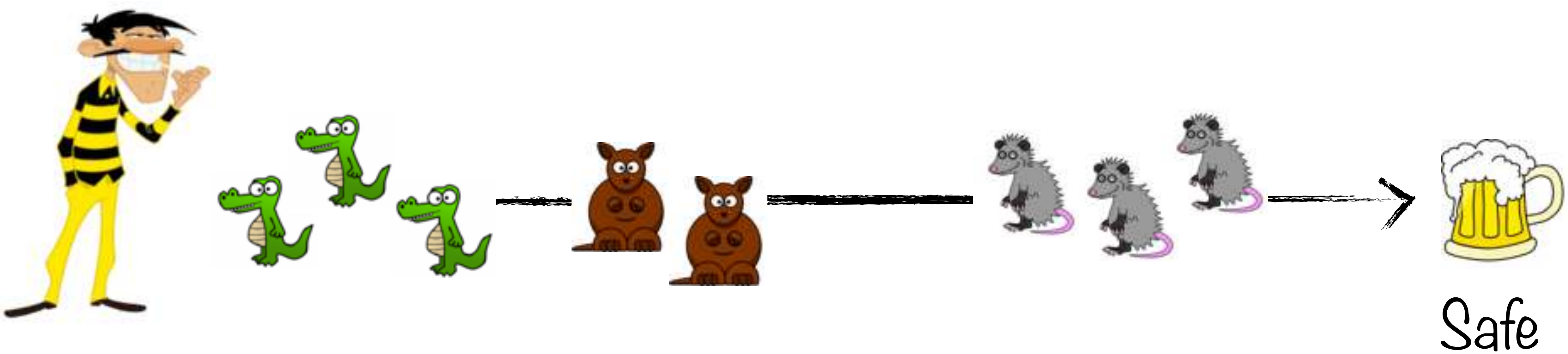


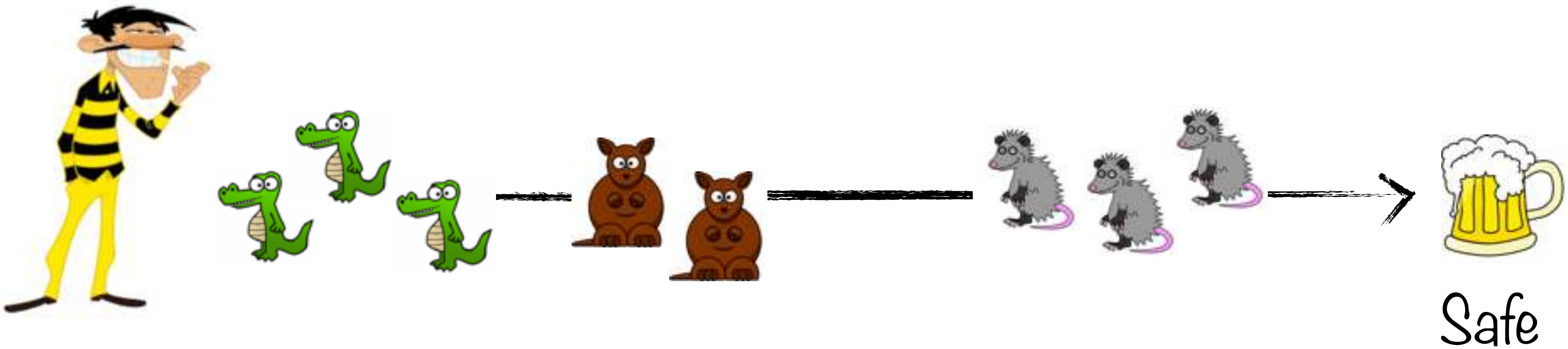
Demo: Uppaal

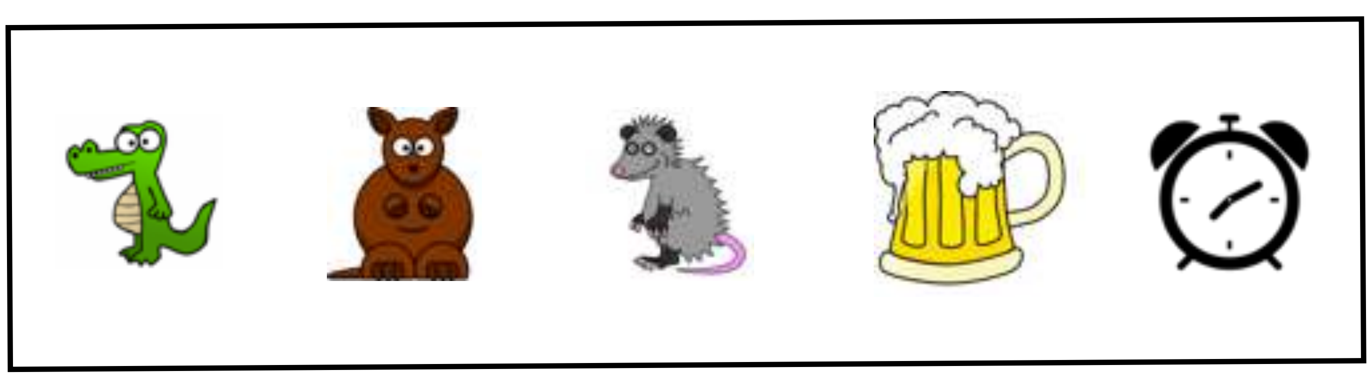
[Larsen et al. 1997]

Symbolic Simulation

Our proposal: keep a notion of time elapsing

Australian Walk





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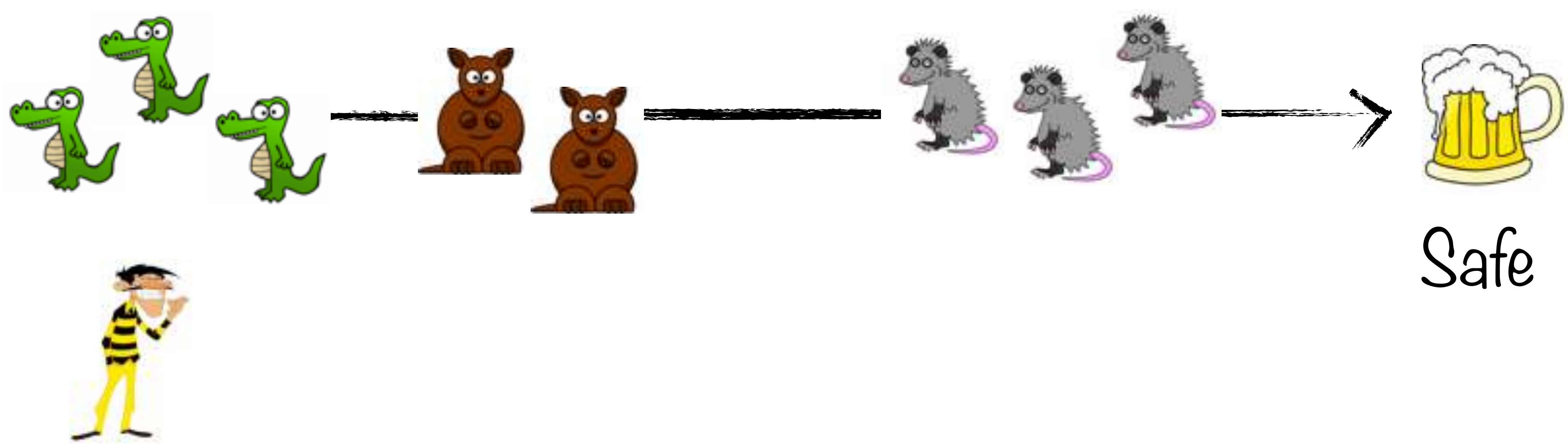

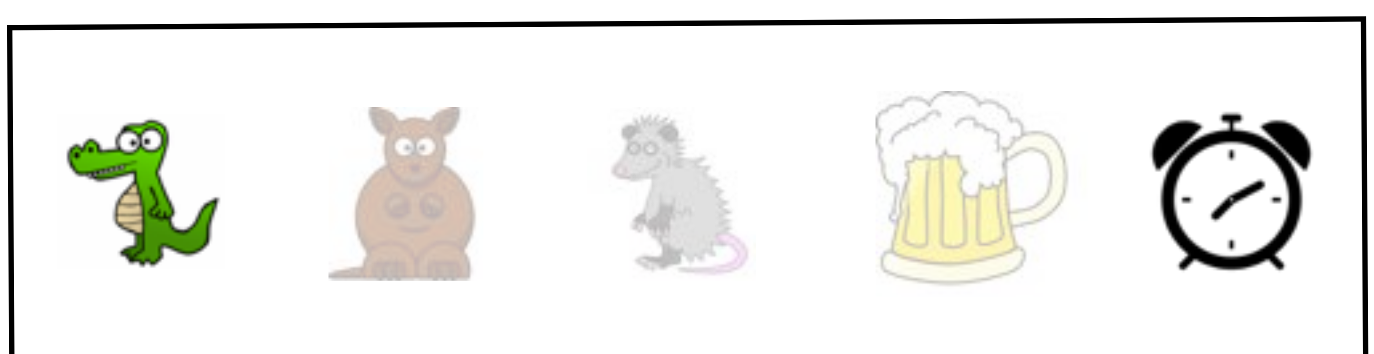

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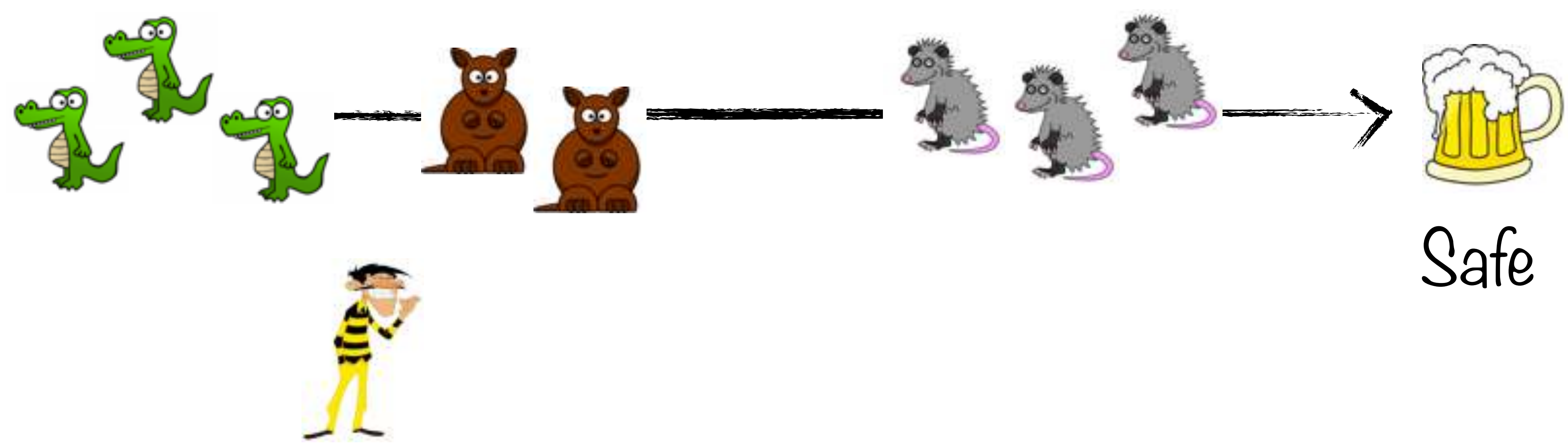

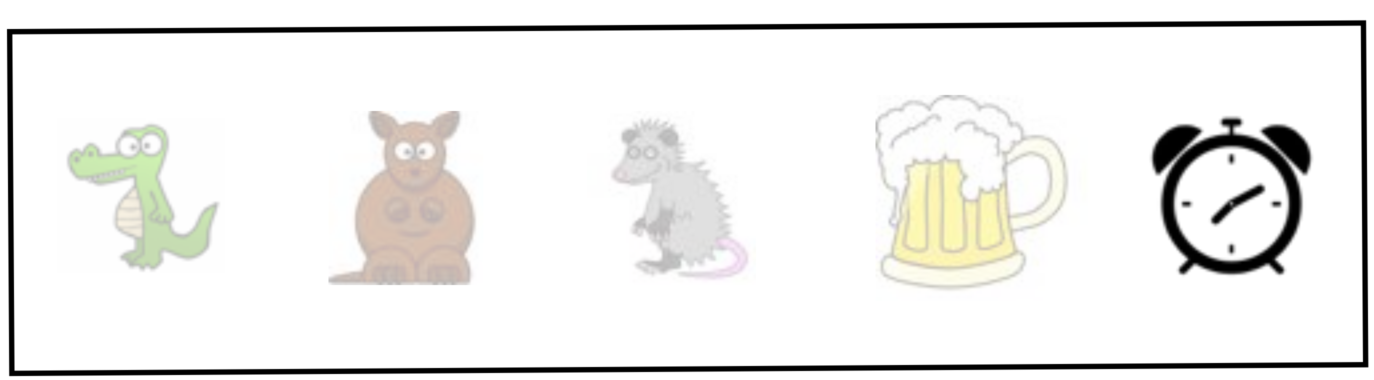

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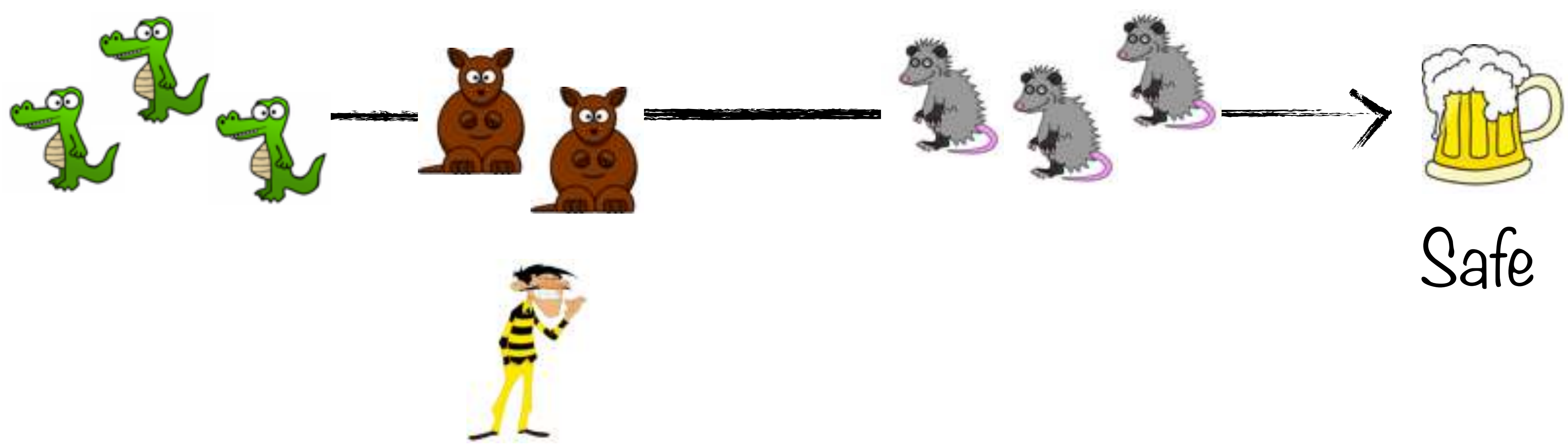
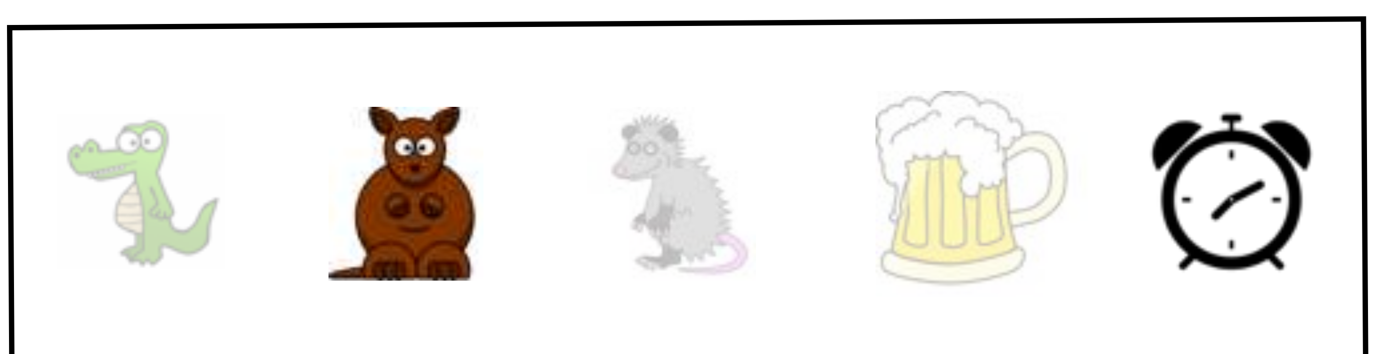
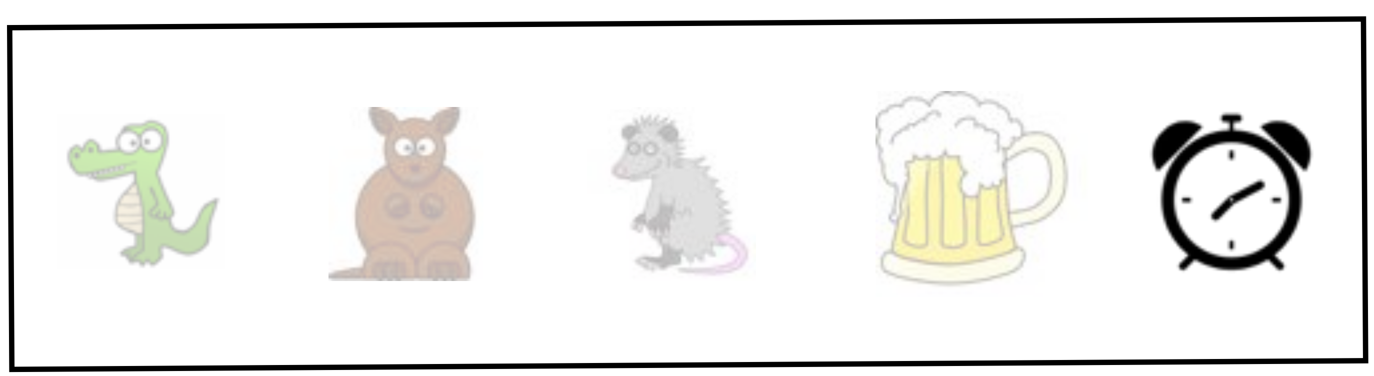




Symbolic Simulation

Our proposal: keep a notion of time elapsing

Australian Walk



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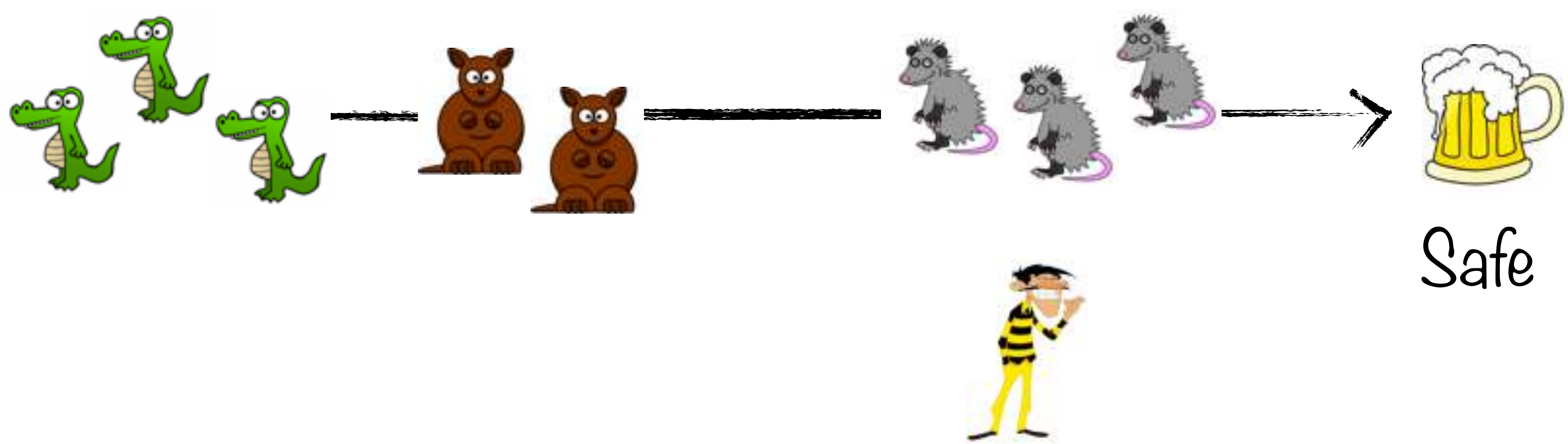

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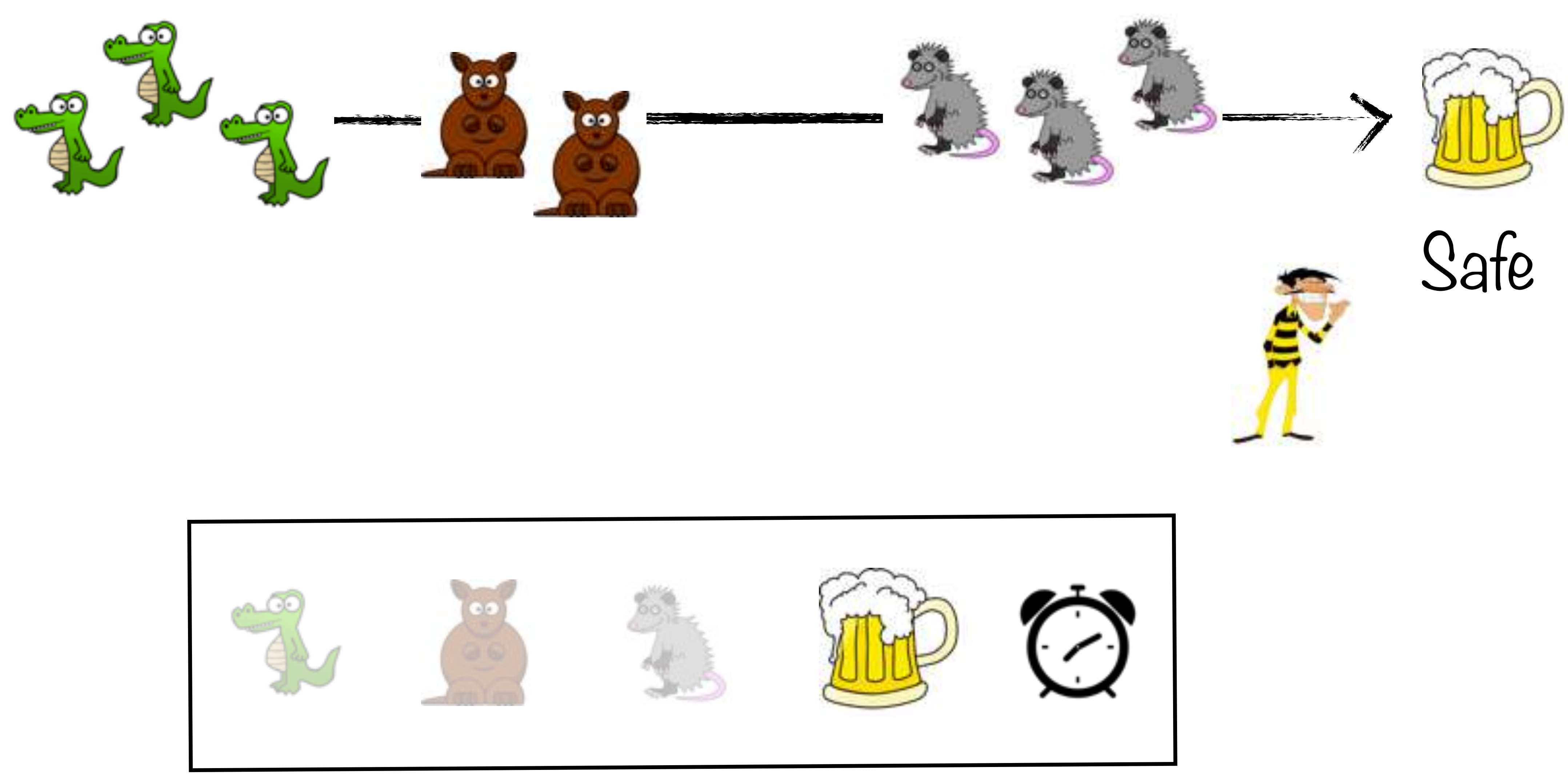

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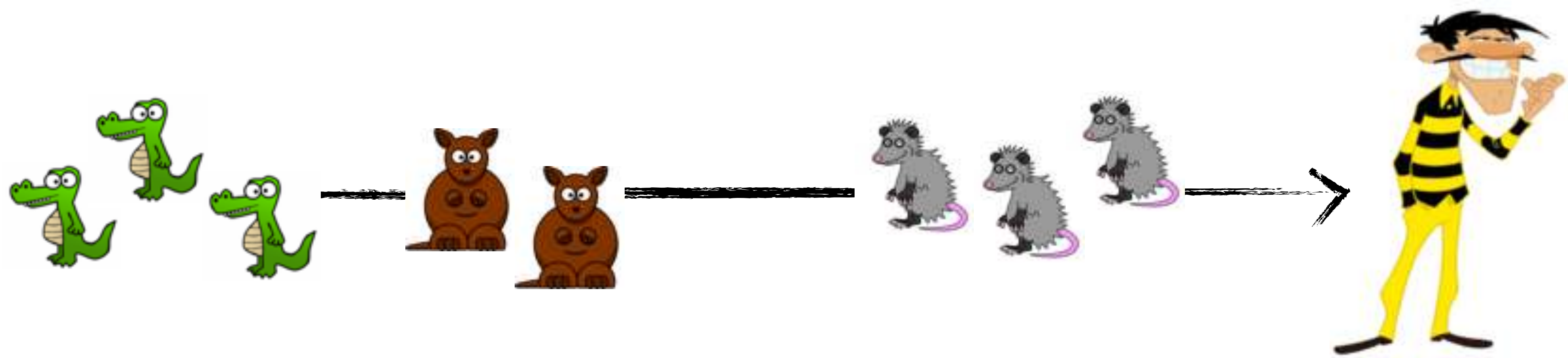
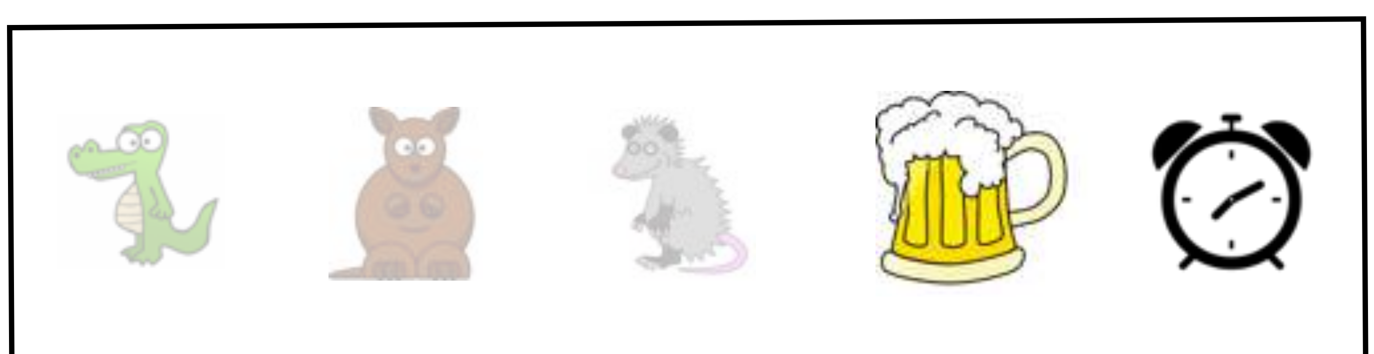
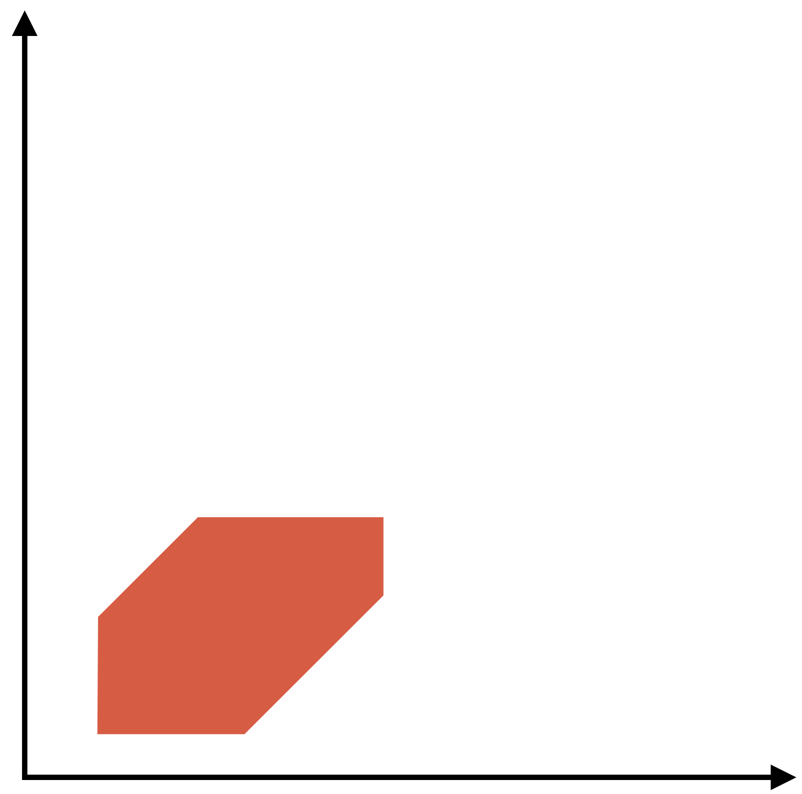




Time Elapse… A New Horizon

Definition: Two simulation states (q,s) and (q',s') are equivalent if q = q' and actions(s) = actions(s')



[Halbwachs et al. 1994]

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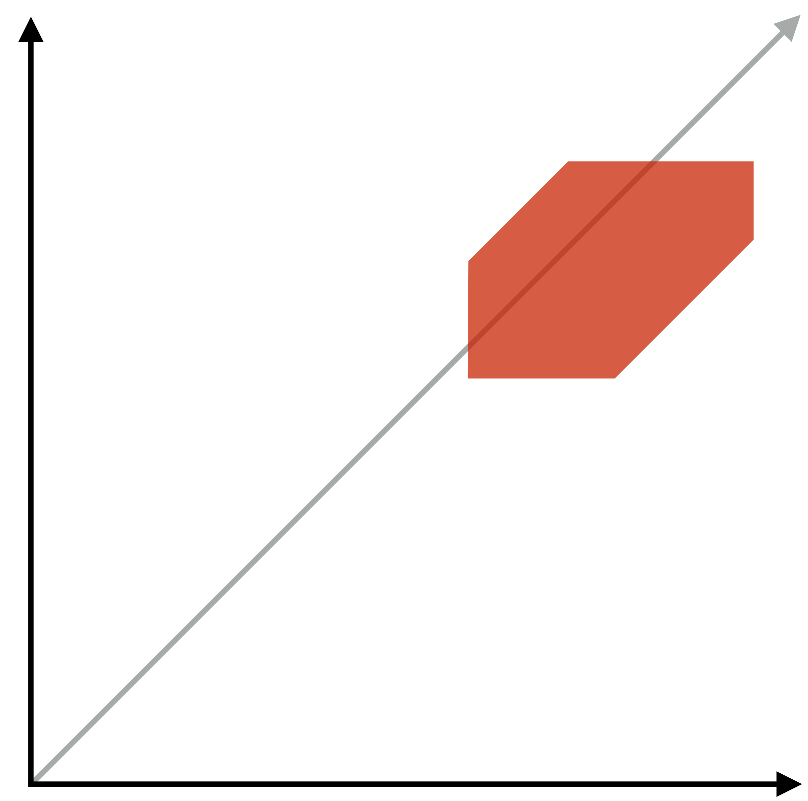

Increment all clocks at the same time: slide along direction (1, 1, …, 1)

[Halbwachs et al. 1994]

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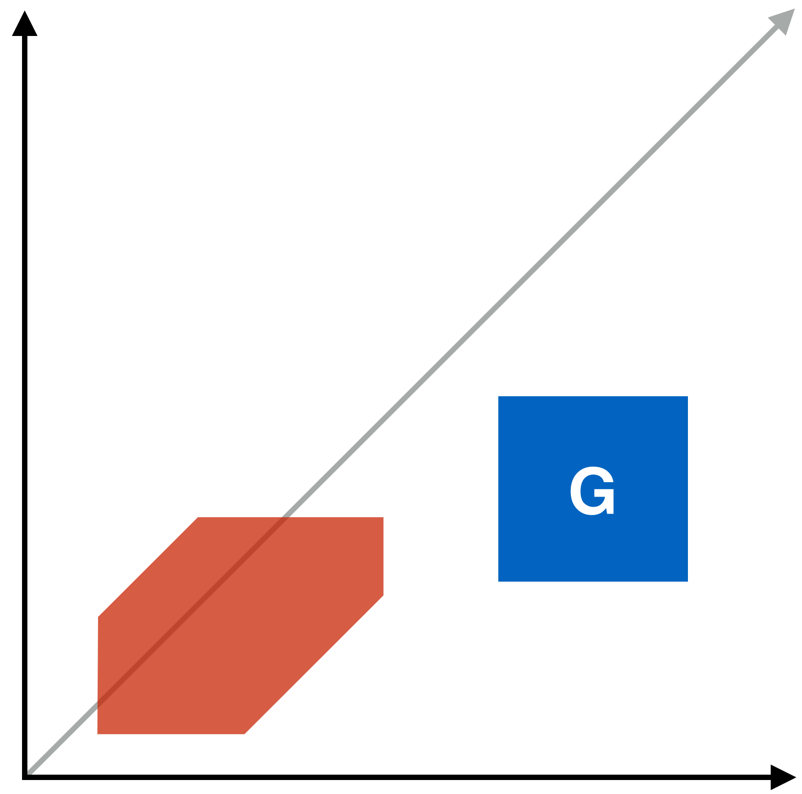

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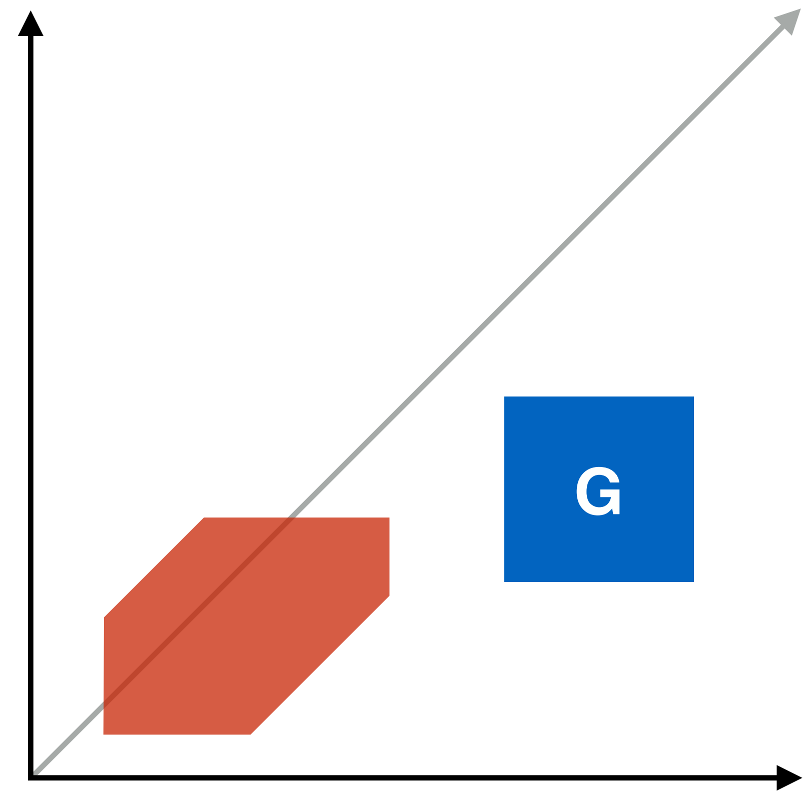

For each action, two horizons: entering and leaving

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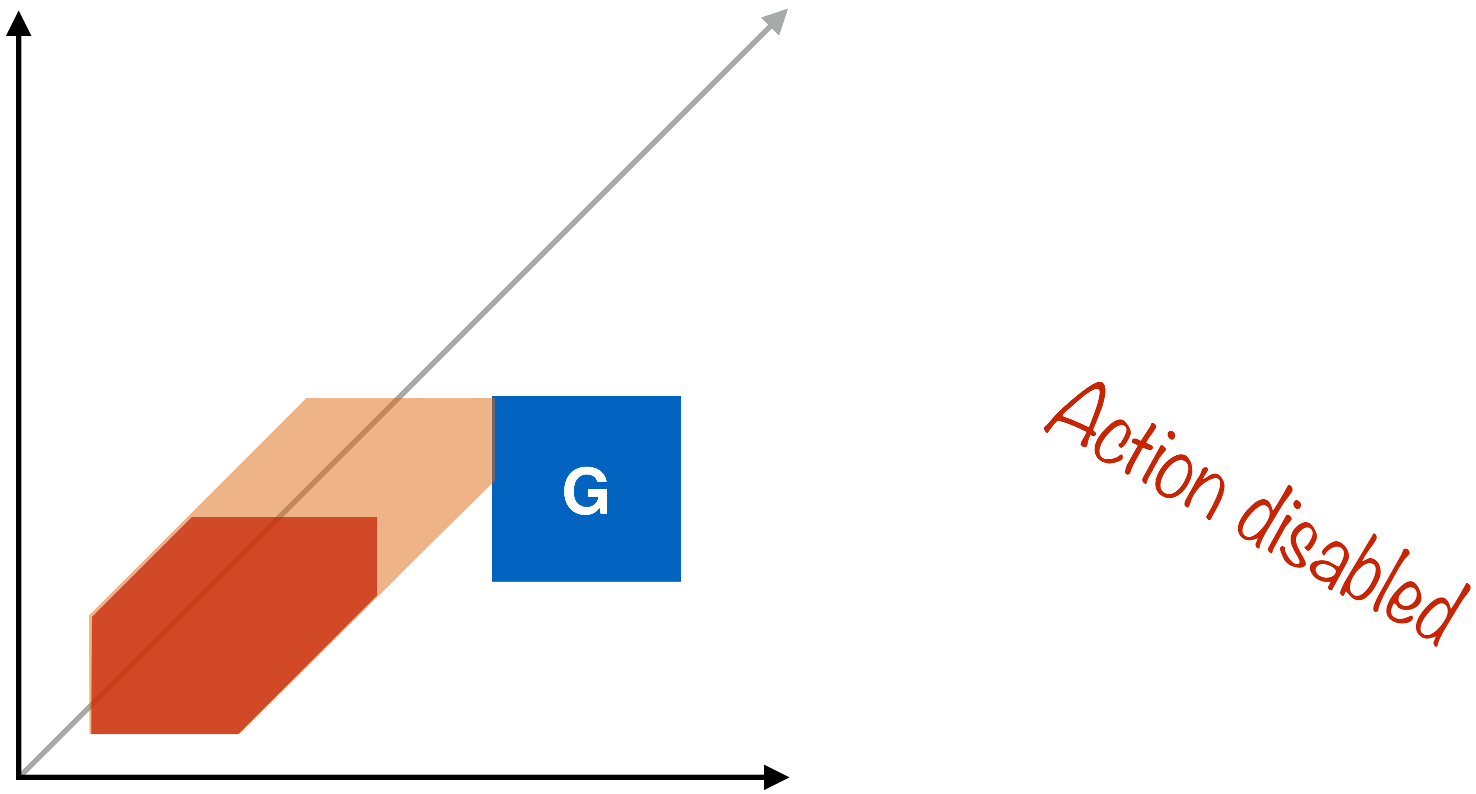

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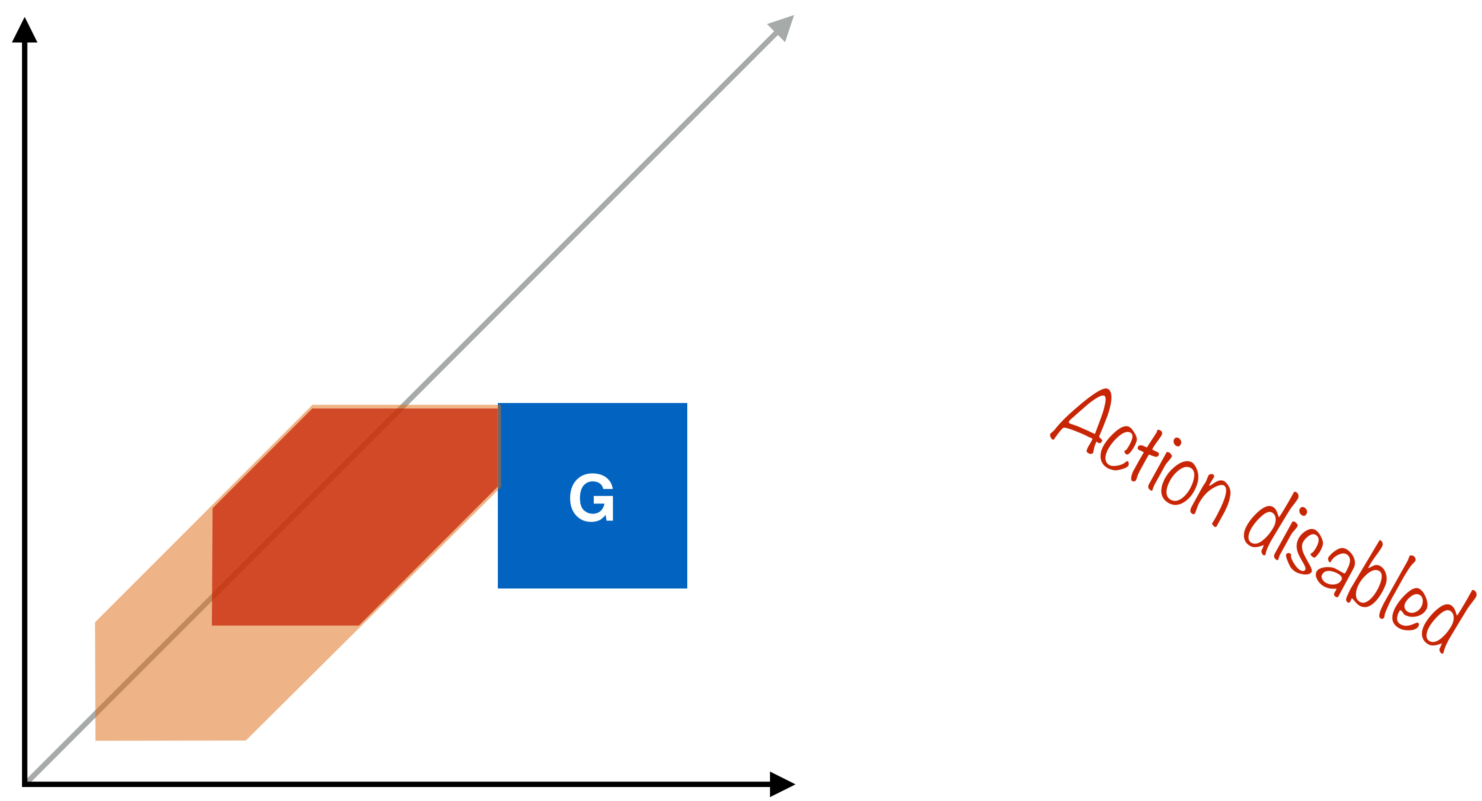

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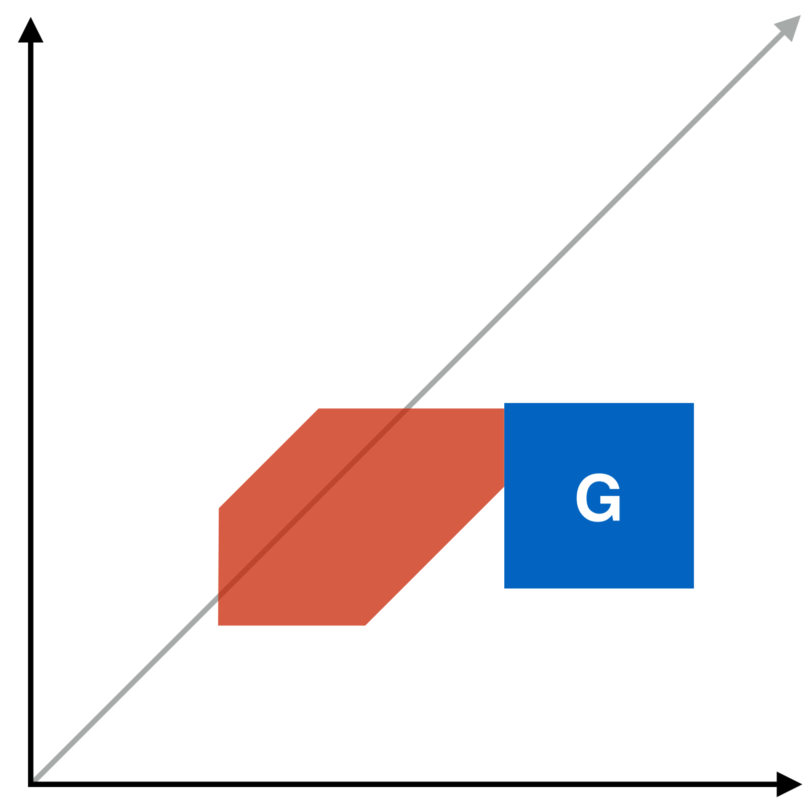

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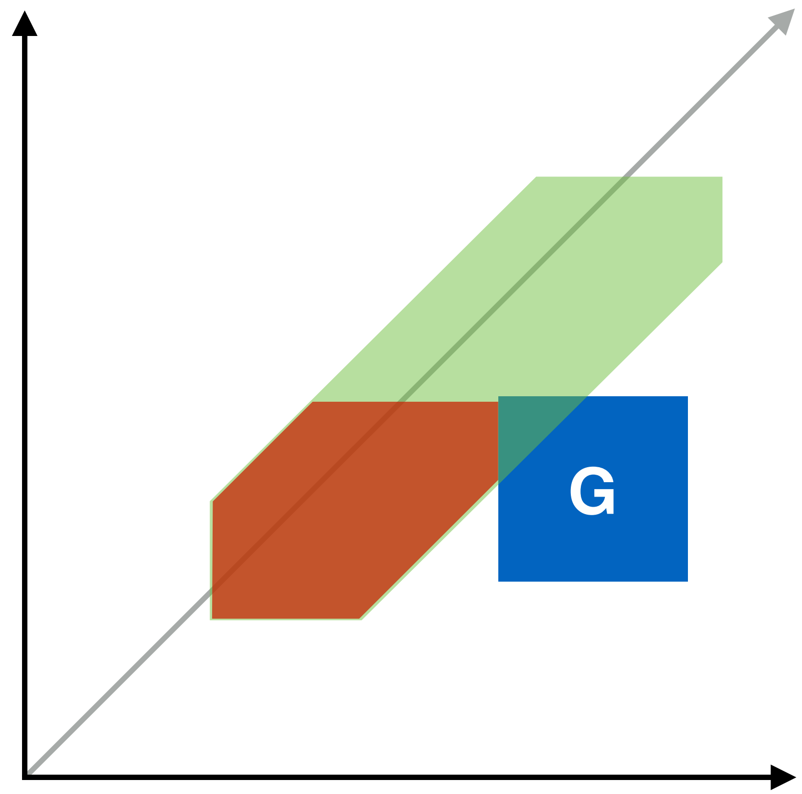

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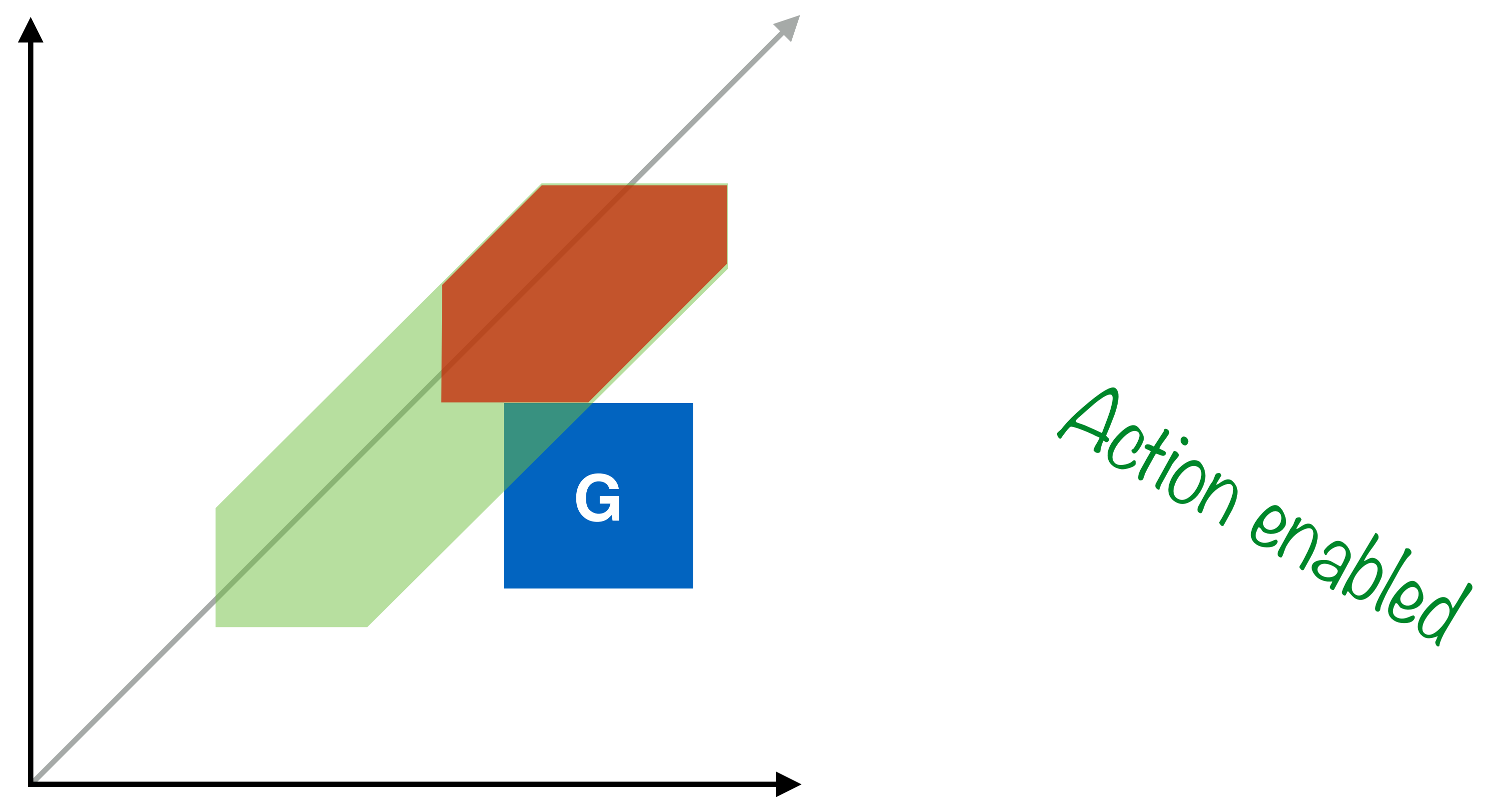

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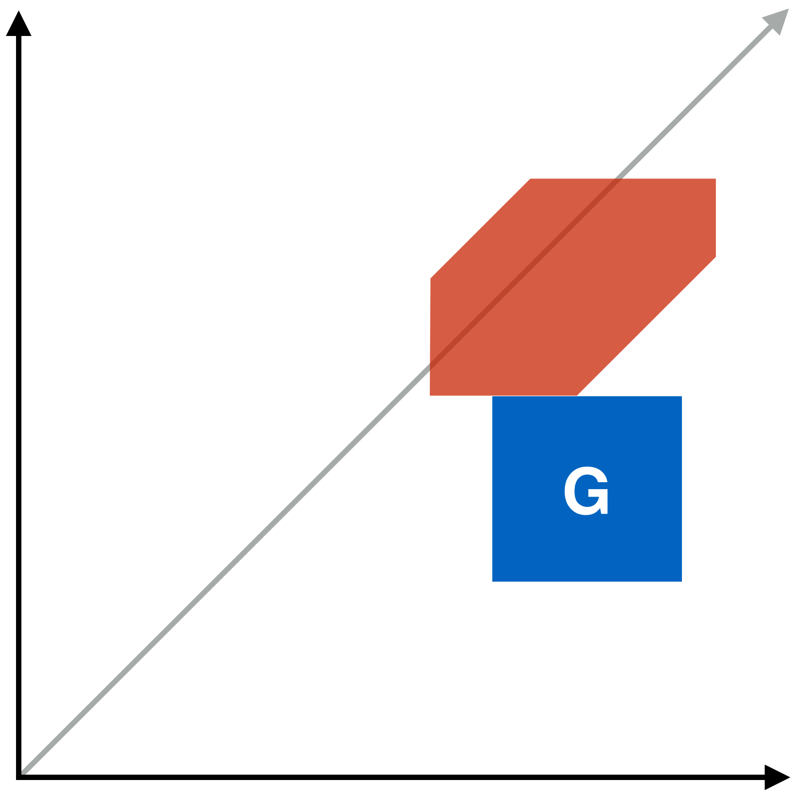

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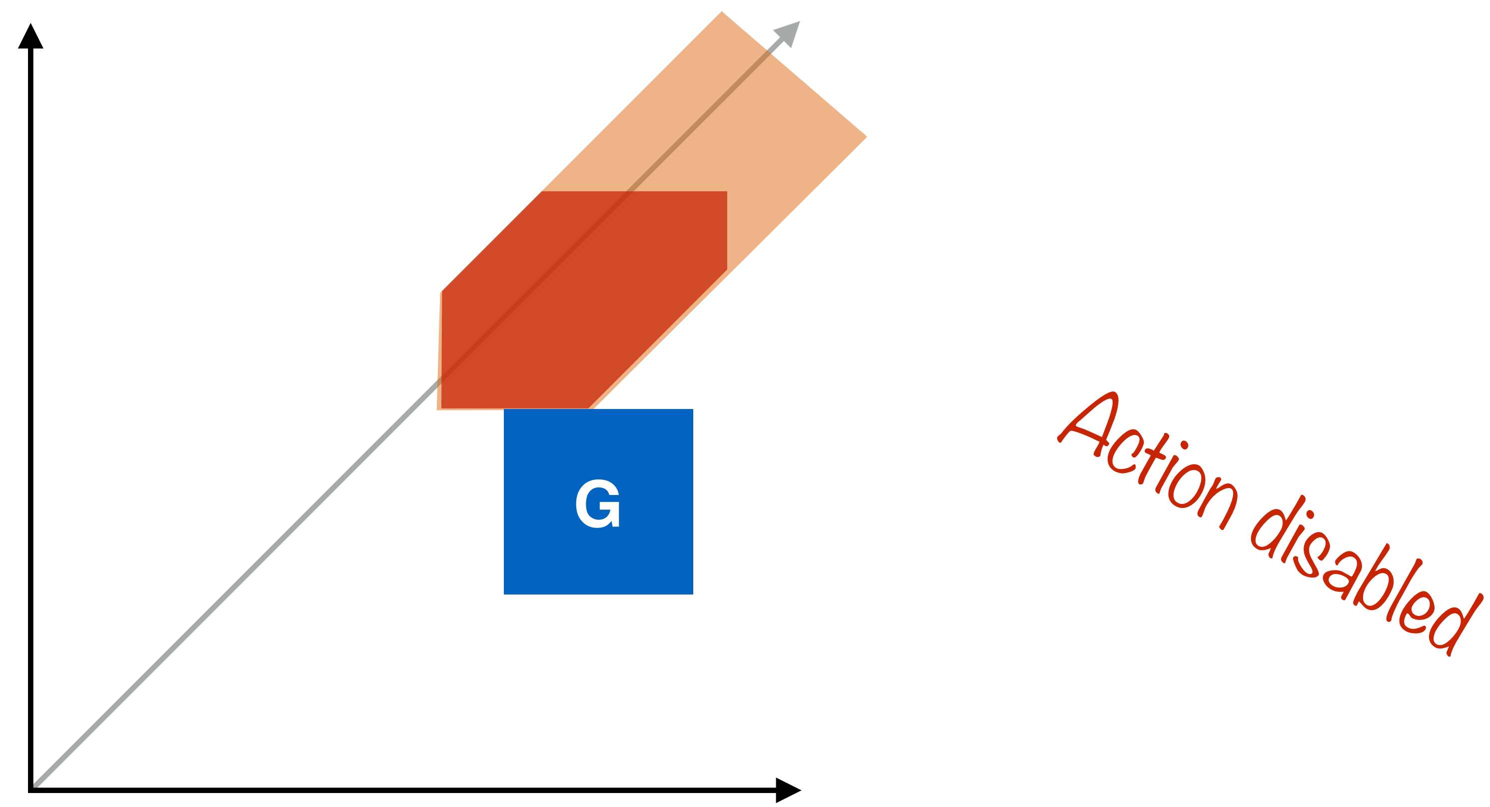

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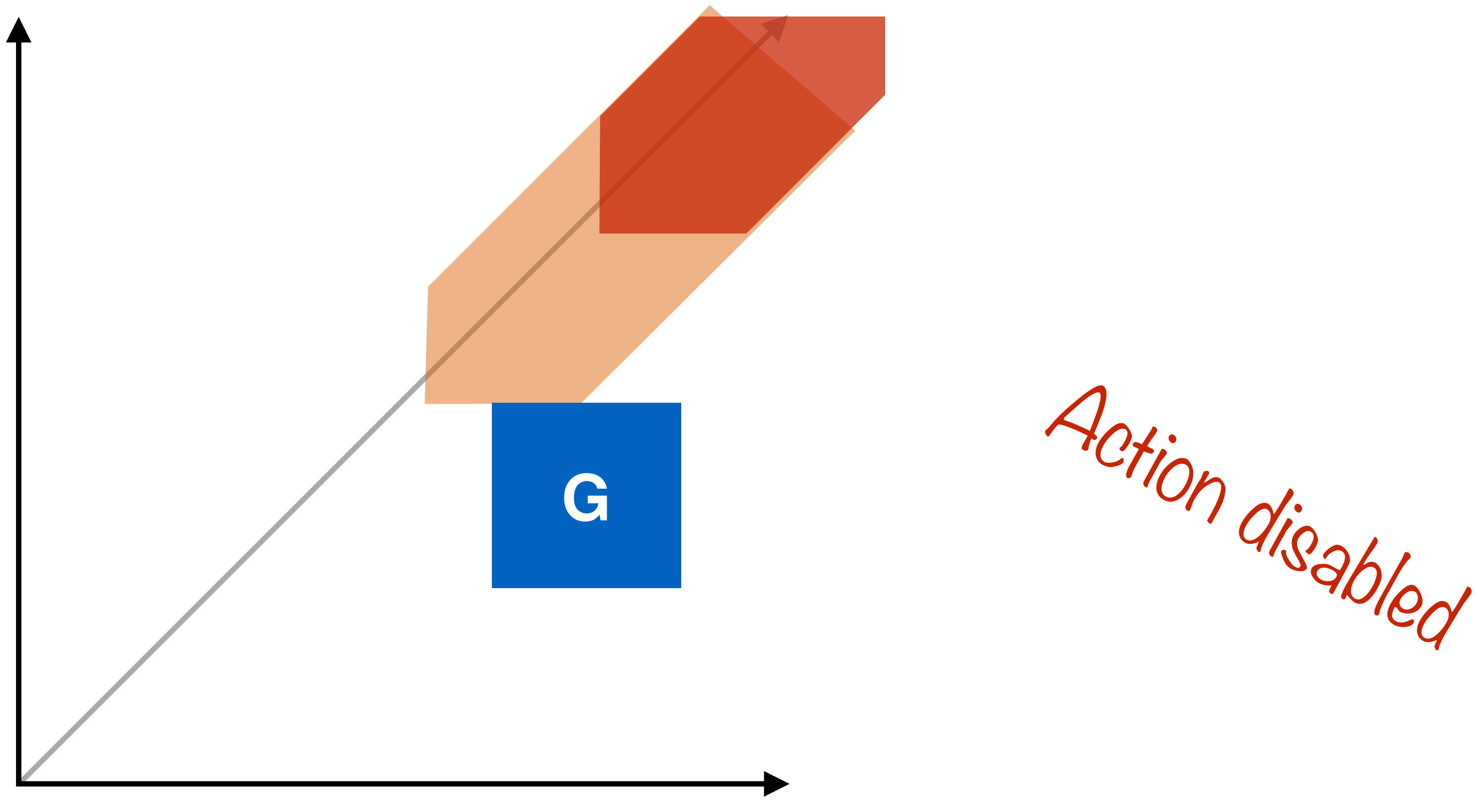

For each action, two horizons: entering and leaving

Language Restrictions

A subset of Zélus

Clocks: `der x = 1.0`

Clock constraints: $(x - y) \bowtie e$

with $\bowtie \in \{\leq, <, >, \geq\}$ and `e: float`

Operations on clocks:

- Reset: `x = v`
- Translation: `x = x + v`
- Synchronization: `x = y`

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Difference Bound Matrices (DBM)

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Clock constraints: `(x - y)` $\bowtie$ `e`

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Operations on clocks:

- Reset: `x = v`
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- Synchronization: `x = y`

[Dill 1990]

Compilation

Modularity: Each block returns the guard of the enabled transitions.

Global State: A DBM represents clock constraints of the entire system (current clock domain).

Simulation: At each step, we return the next horizon and compute the next clock domain.

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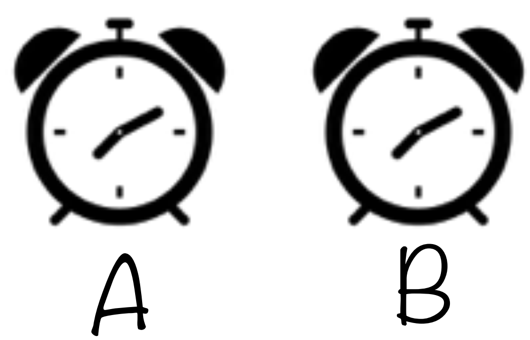
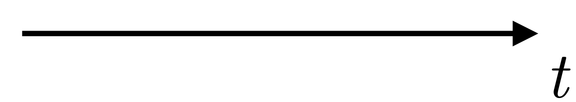
A way to discretize continuous systems

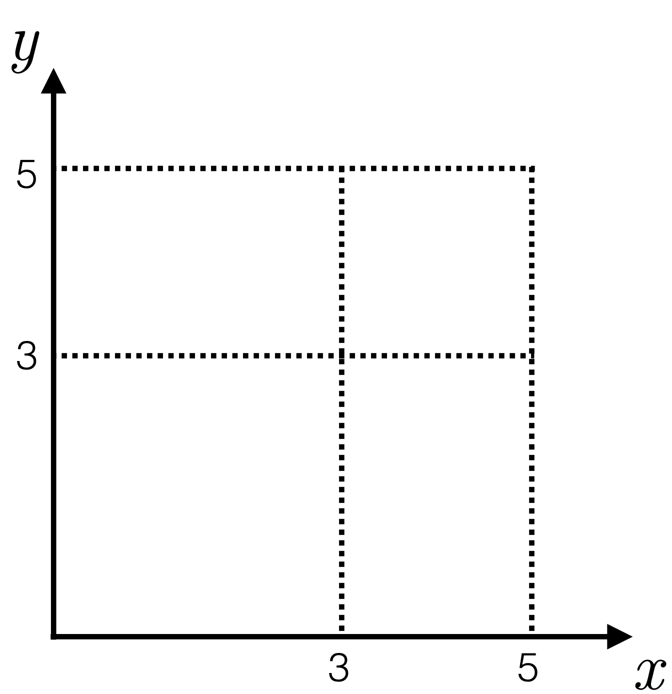
Example: Quasi-Periodic Architectures

Symbolic Simulation of a pair of quasi-periodic clocks?

```
let hybrid qp_archi (in_x, in_y) = ticA, ticB where
  rec ticA = metro (in_x, 3, 5)
  and ticB = metro (in_x, 3, 5)
```







in_x

in_y

wait

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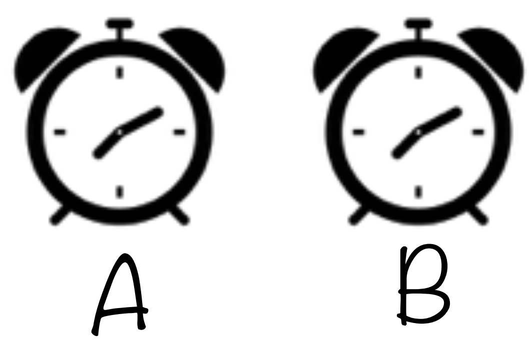
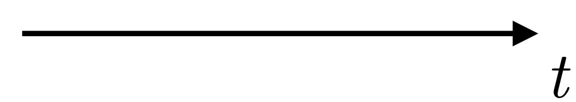
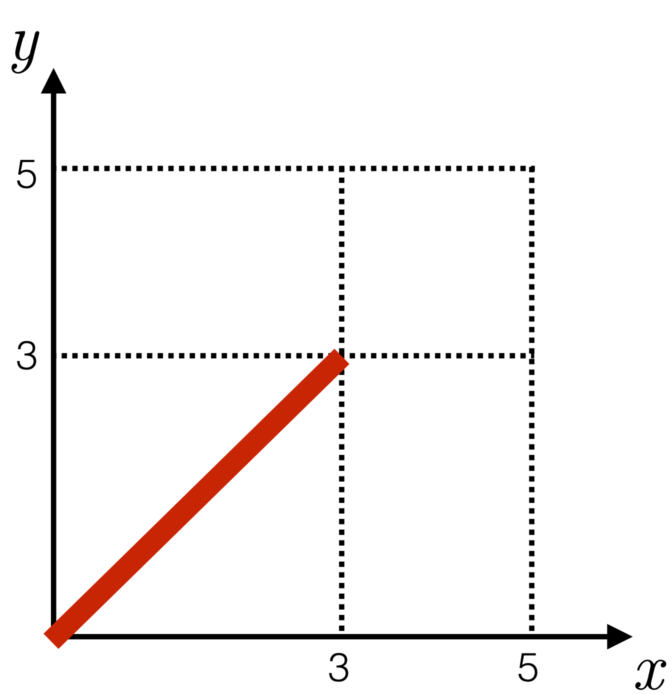
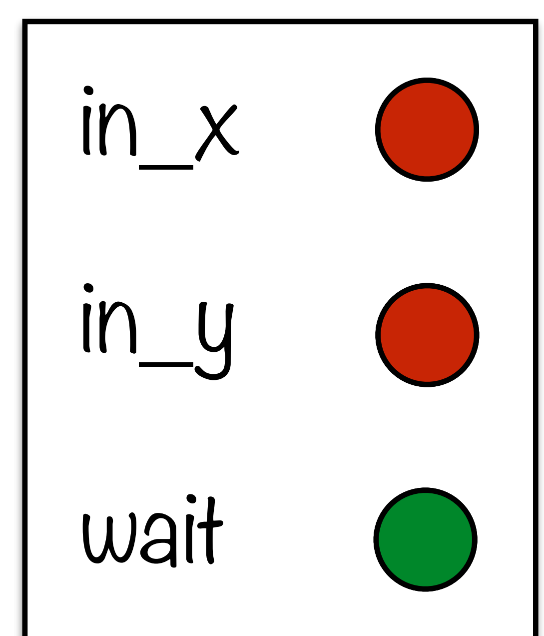






$$\left\{\begin{array}{l} 0 \leq x < 3 \\ 0 \leq y < 3 \\ x - y = 0 \end{array}\right\}$$



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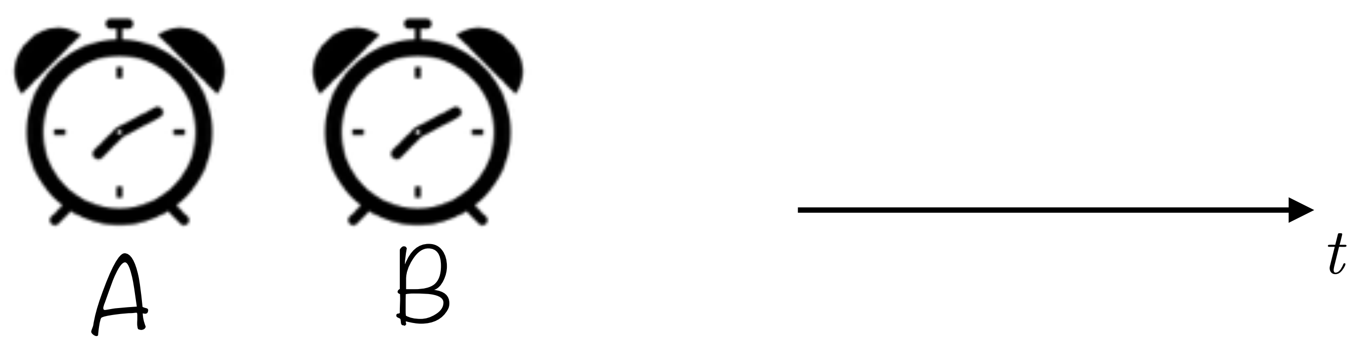
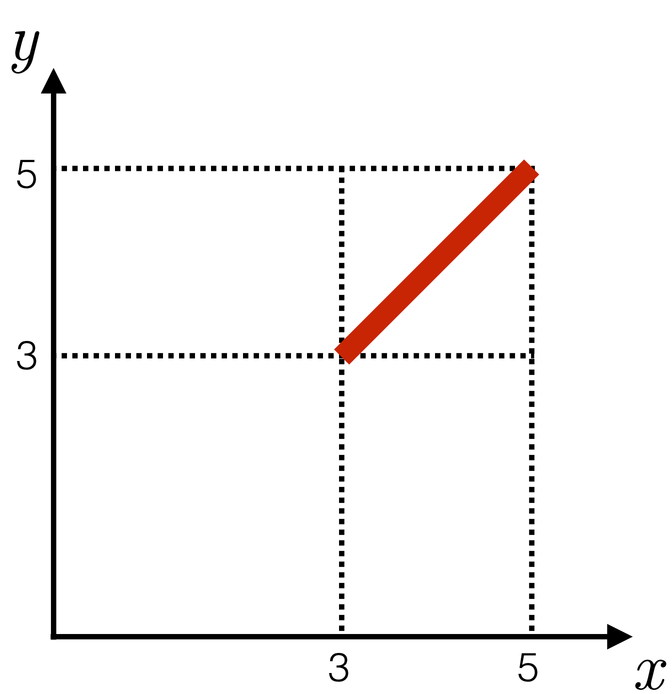
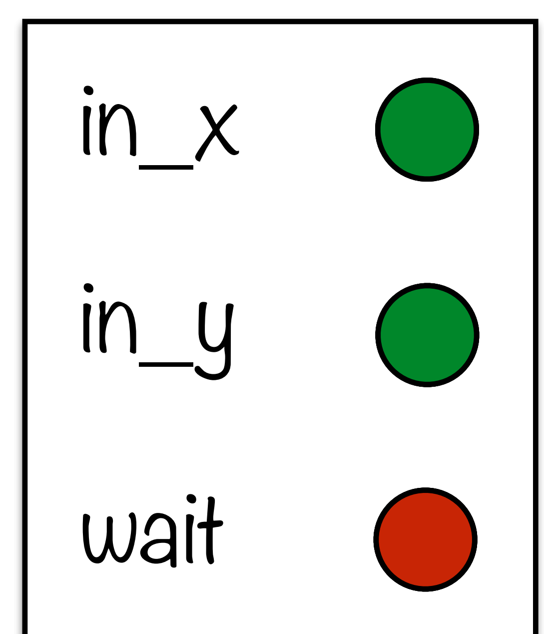




$$\begin{Bmatrix} 3 \leq x \leq 5 \\ 3 \leq y \leq 5 \\ x - y = 0 \end{Bmatrix}$$



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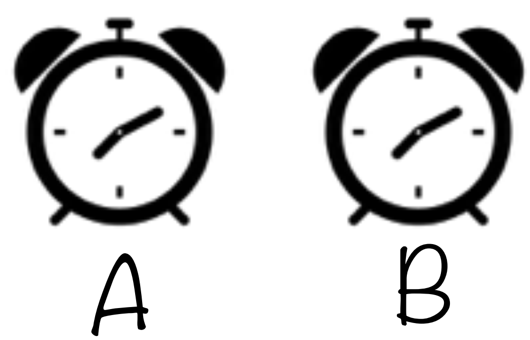
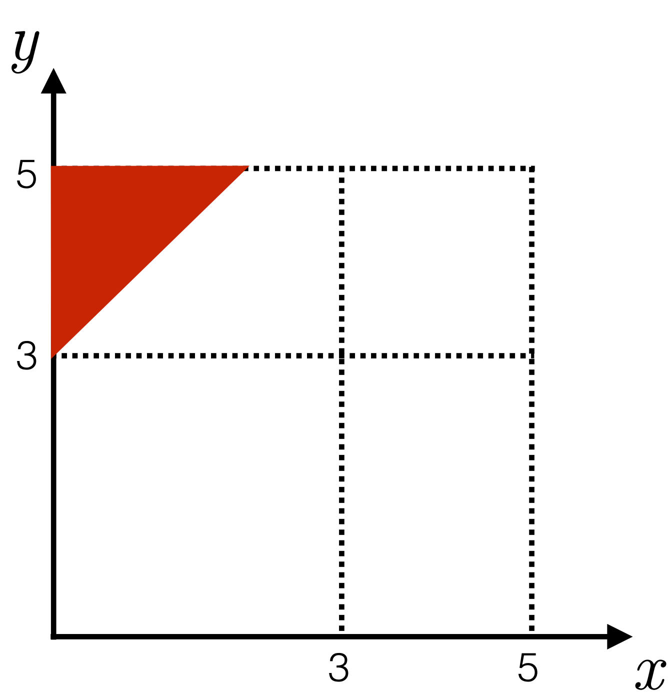
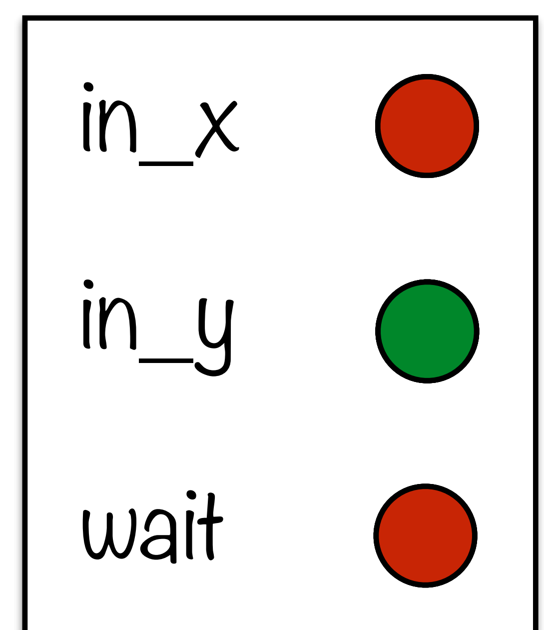




$$\left\{ \begin{array}{c} 0 \leq x < 2 \\ 3 \leq y \leq 5 \\ 3 \leq y - x \leq 5 \end{array} \right\}$$



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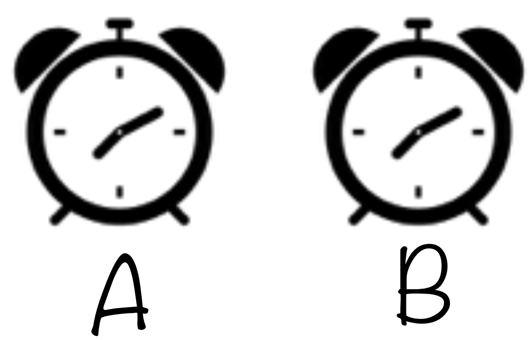
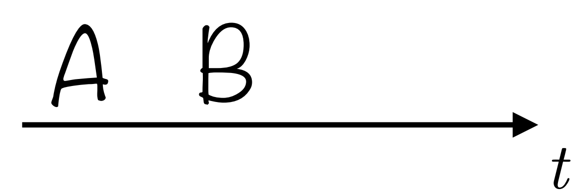
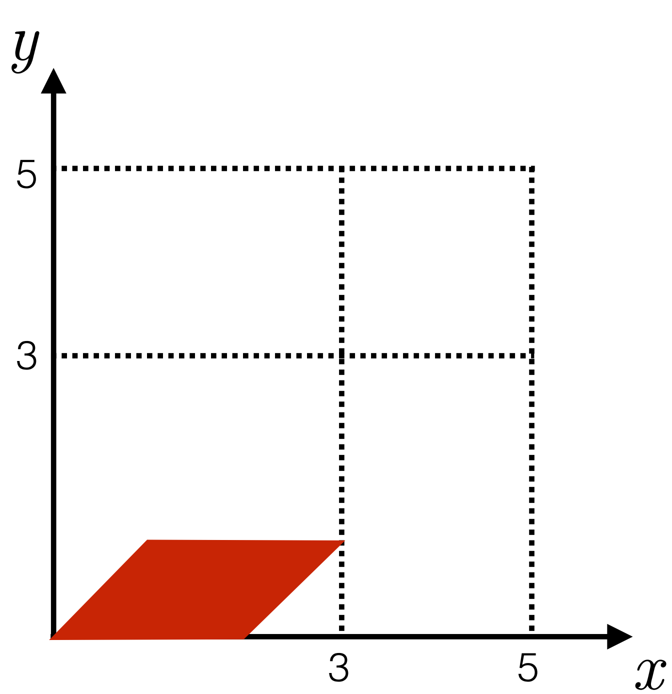
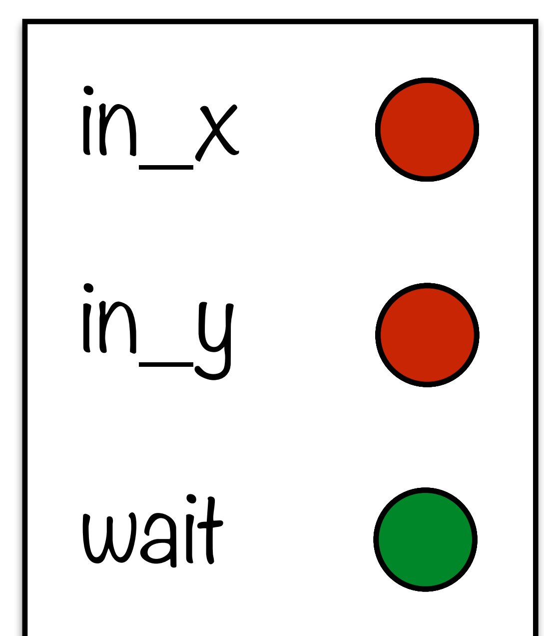






$$\left\{ \begin{array}{c} 0 \leq x < 3 \\ 0 \leq y \leq 1 \\ 0 \leq x - y \leq 2 \end{array} \right\}$$



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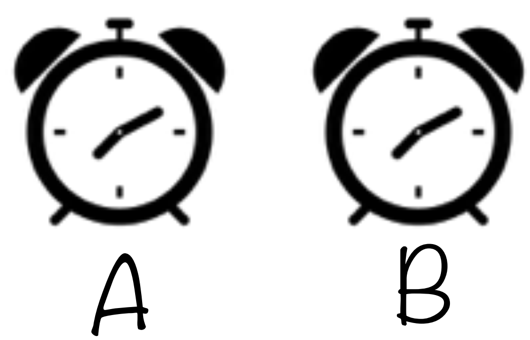
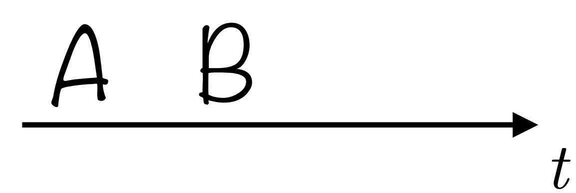
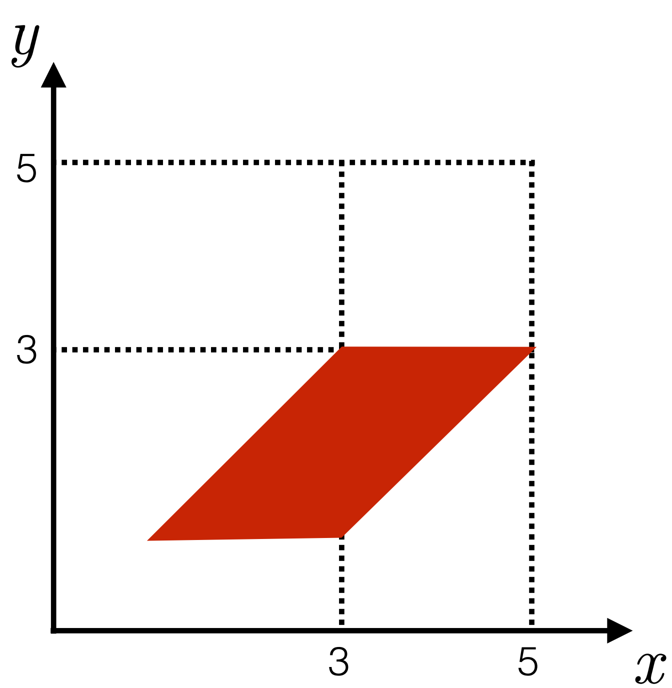
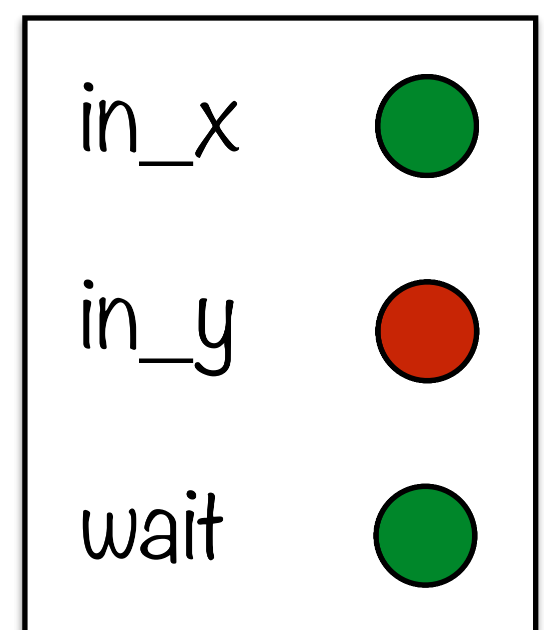






$$\left\{\begin{array}{c} 1 \leq x \leq 5 \\ 1 \leq y < 3 \\ 0 \leq x - y \leq 2 \end{array}\right\}$$



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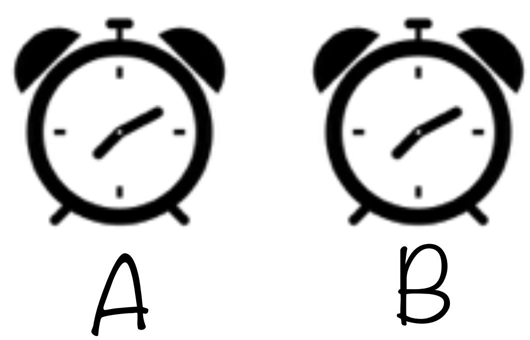

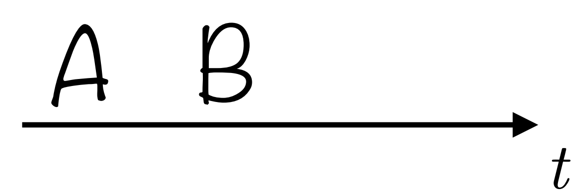

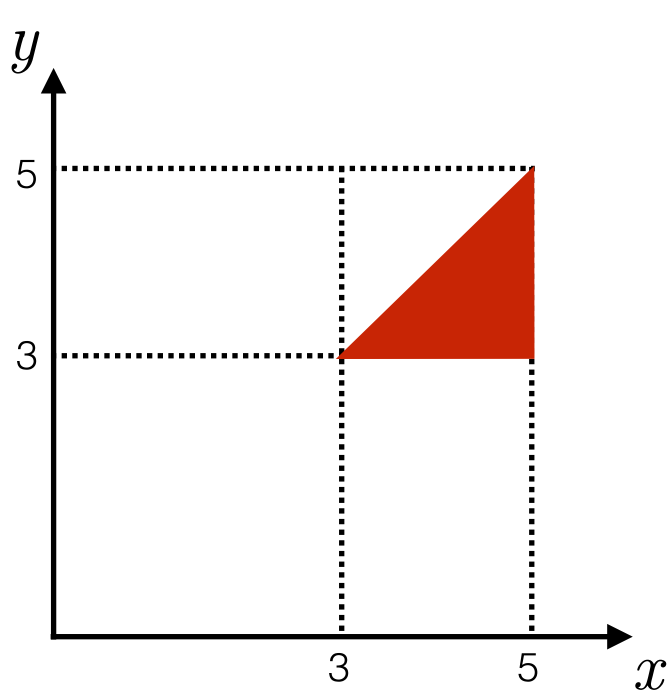

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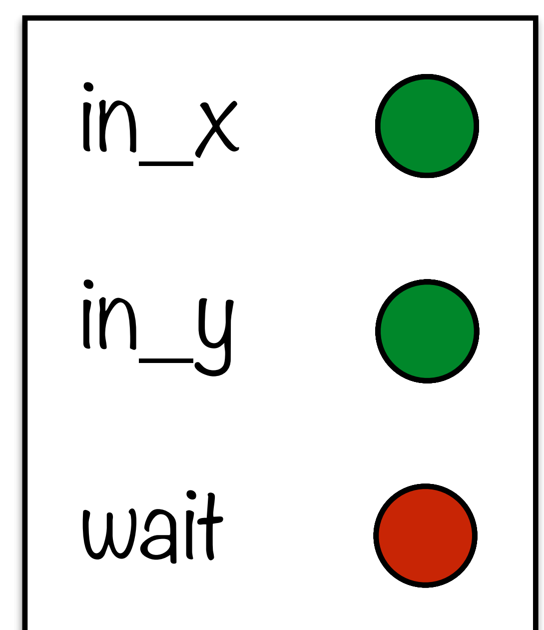

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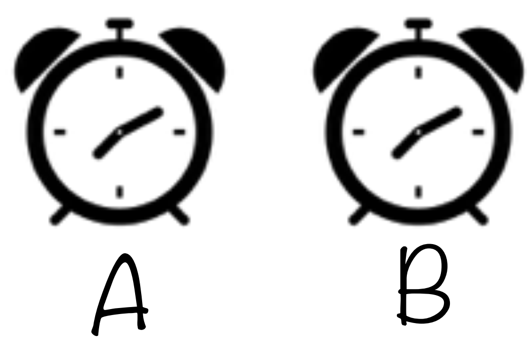

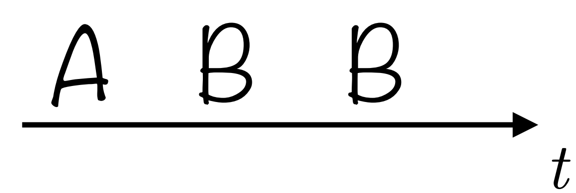

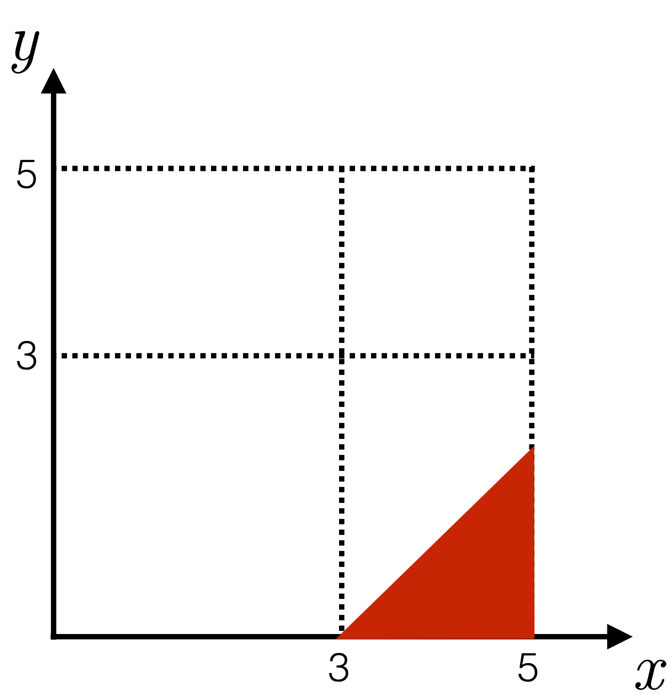

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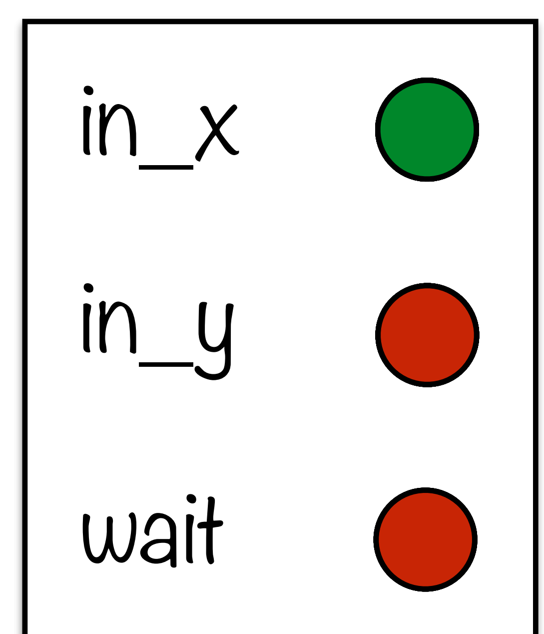

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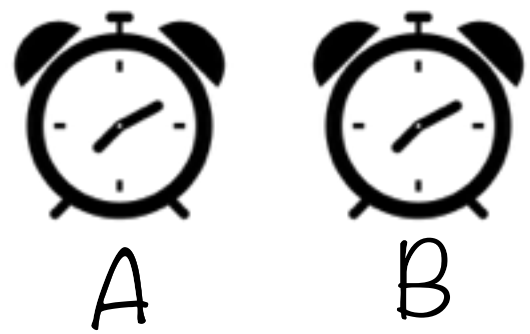
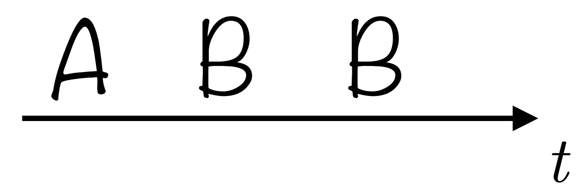
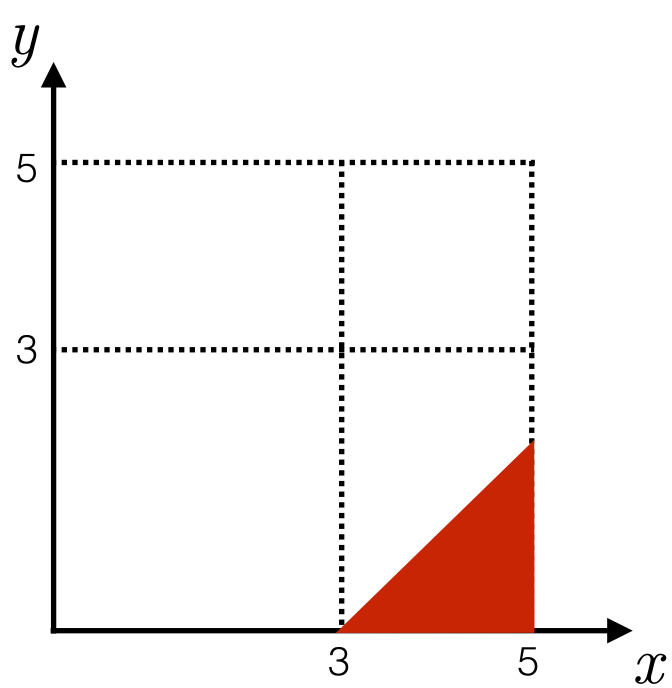
No more than two ticks of one clock between two ticks of the other







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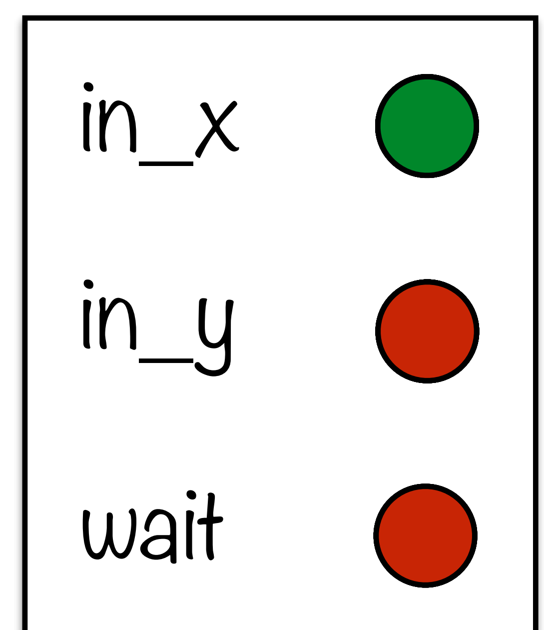

Future Work

Prototype implementation in zélus
Source to source transformation and runtime

More complex clock domains
octagon, polyhedron, …

Under-approximation / Over-approximation
safety vs. precision

Generate discrete controllers
for instance quasi-synchronous controllers

Improve test coverage
see [Alur et al 2008]